CHAPTER 2

RESULTANT OF COPLANAR FORCE SYSTEMS

2–1 INTRODUCTION

Two systems of forces are said to be *equivalent* if they produce the same mechanical effect on a rigid body. A single force that is equivalent to a given force system is called the *resultant* of the force system.

We shall first introduce the *parallelogram law* and use it to find the resultant of concurrent coplanar forces. Then the rectangular components of forces are discussed and used to find the resultants.

As we shall see later, any system of nonconcurrent coplanar forces can be replaced by a *single resultant* that is equivalent to the given force system. The location of the line of action of the resultant is not immediately known. To determine the line of action of the resultant of a nonconcurrent coplanar force system, we will introduce the concepts of the *moment* of a force first. The resultant of some simple types of distributed line loads will be discussed in this chapter also.

2–2 VECTOR REPRESENTATION

Notations. In this book, vector quantities will be distinguished from scalar quantities through the use of boldface type, such as **P.** An italic type, such as *P*, will be used to denoted the magnitude of a vector. In long-hand writing, vectors may be represented by the notation $\overrightarrow{\mathrm{P}}$.

Graphical Representation. A force **F** (or any vector quantity) is represented graphically by a line segment *AB* with an arrowhead at one end, as shown in Fig. 2–1. *A* is the point of application and *x* is a reference coordinate axis. The length of the line segment *AB* represents the magnitude of the force measured according to some convenient scale. The direction is indicated by the angle θ from the reference axis.

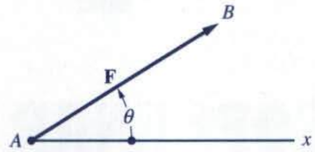


FIGURE 2–1

Equal Vectors. Two vectors having the same magnitude and the same direction are said to be equal. Two equal vectors may or may not have the same line of action (Fig. 2–2). Equal vectors may be denoted by the same letter.

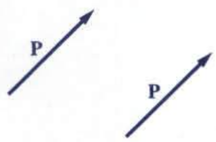


FIGURE 2–2

Negative Vector. The negative vector of a given vector **P** is defined as a vector having the same magnitude as **P** and a direction opposite to that of **P** (Fig. 2–3). The negative vector of **P** is denoted by −**P**. According to Newton's law of action and reaction presented in Section 1–6, the forces of action and reaction must always be equal in magnitude and opposite in direction. Thus, the forces of action and reaction may be represented by **P** and −**P**.

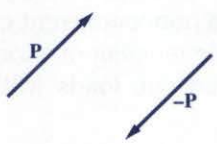


FIGURE 2–3

2–3 RESULTANT OF CONCURRENT FORCES

Parallelogram Law. As mentioned in Section 1–3, vectors are added according to the *parallelogram law.* Figure 2–4 shows two vectors that are added according to this law. The two vectors **P** and **Q** are placed at the same point *A* and a parallelogram is constructed using **P** and **Q** as its two adjacent sides. The diagonal of the parallelogram from *A* to the opposite corner represents the sum of **P** and **Q** and is denoted by **P** + **Q**. Note that, in general, the magnitude of the vector sum **P** + **Q** is not equal to the algebraic sum of the magnitudes *P* and *Q*.

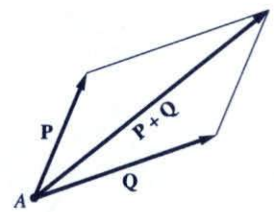


FIGURE 2–4

Triangle Rule. The sum of two vectors can also be determined by constructing one-half of the parallelogram, or a triangle. This method is called the *triangle rule.* To find the vector sum **P** + **Q,** we first lay out **P** at *A* (Fig. 2–5a), then lay out **Q** from the tip of **P** in a tip-to-tail fashion. The closing side of the triangle, drawn from *A* to the tip of **Q,** represents the sum of the two vectors. Figure 2–5b shows that the same result is obtained if the vector **Q** is laid out first. Hence, the vector sum is not affected by the order in which the vectors are added; that is, *vector addition is commutative:*

$$\mathbf{P} + \mathbf{Q} = \mathbf{Q} + \mathbf{P} \tag{2–1}$$

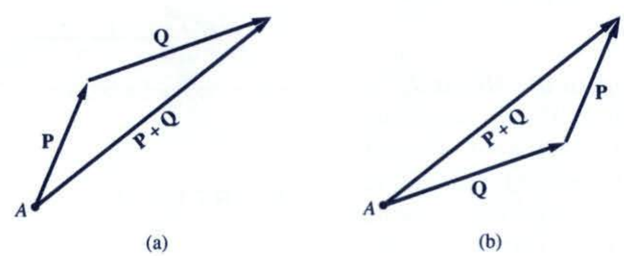


FIGURE 2–5

Polygon Rule. The sum of three or more concurrent coplanar vectors may be carried out by adding two vectors successively. For example, the sum of three coplanar concurrent vectors, **P, Q,** and **S** (Fig. 2–6a) can be obtained by first finding **P** + **Q,** then adding **S** to **P** + **Q** to find **P** + **Q** + **S,** as shown in Fig. 2–6b. Notice that the dotted line in the figure could be omitted, and the sum of the vectors can be obtained directly by laying out the given vectors in a tip-to-tail fashion to form the sides of a polygon. The closing side of the polygon, from the starting point to the final point, represents the sum of the vectors. This is known as the *polygon rule* for the addition of vectors. A polygon formed by forces is called a *force polygon.*

Since the vector sum is commutative, *the order in which the vectors are added is arbitrary.* In Fig. 2–6c the vectors are added in the order of **P, S,** and **Q.** We see that, although the shape changes, the resultant obtained remains the same.

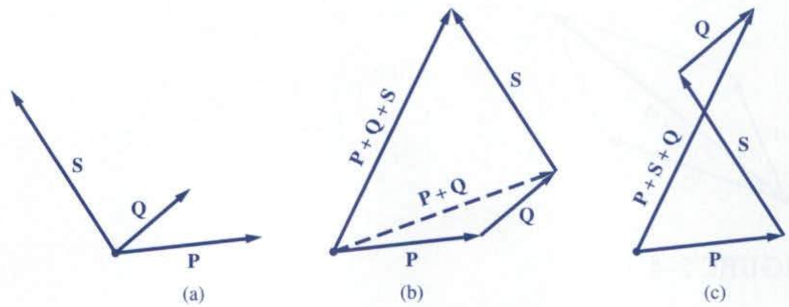


FIGURE 2–6

Resultant. A given system of concurrent coplanar forces acting on a rigid body may be replaced by a single force, called the *resultant,* equal to the vector sum of the given forces. The resultant will produce the same effect on the rigid body as the given force system.

EXAMPLE 2–1

Determine the resultant of two forces **P** and **Q** acting on the hook in Fig. E2–1(1).

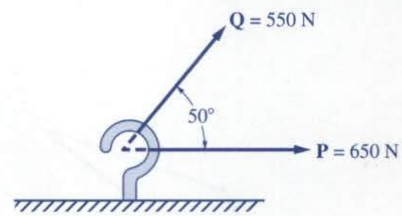


FIGURE E2–1(1)

Solution. Two methods are presented here.

(a) The Graphical Method. A force triangle *ABC* is drawn as shown in Fig. E2–1(2). *AB* represents force **P** and *BC* represents force **Q.** The magnitude of each force is laid out by using a properly chosen linear scale. The direction of each force is measured by using a protractor. The closing side of the triangle, *AC,* is the resultant. The magnitude and direction of the resultant are measured to be

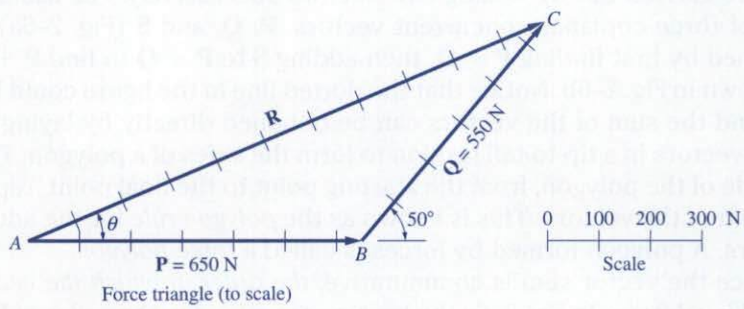


FIGURE E2–1(2)

$$R = 1090 \text{ N} \qquad \theta = 23^\circ$$

$$\mathbf{R} = 1090 \text{ N} \measuredangle\ 23^\circ \qquad \Leftarrow \textbf{Ans.}$$

(b) ***The Trigonometric Method.*** First, the force triangle *ABC* is sketched. [See Fig. E2–1(3).] A freehand sketch is usually sufficient for this purpose. The triangle has two known sides and a known angle between the two sides. The magnitude of the resultant can be computed by applying the law of cosines.

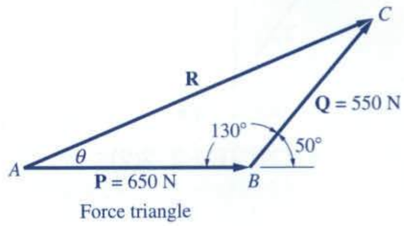


FIGURE E2–1(3)

$$\begin{aligned} R &= \sqrt{P^2 + Q^2 - 2PQ\cos B} \\ &= \sqrt{(650\text{ N})^2 + (550\text{ N})^2 - 2(650\text{ N})(550\text{ N})\cos 130^\circ} \\ &= 1088\text{ N} \end{aligned}$$

To find the direction of the resultant, we compute angle $A(\theta)$ by applying the law of sines.

$$\begin{aligned} \frac{\sin A}{Q} &= \frac{\sin B}{R} \\ A &= \sin^{-1}\left(\frac{Q\sin B}{R}\right) \\ &= \sin^{-1}\left(\frac{550\sin 130^\circ}{1088}\right) \\ &= 22.8^\circ \end{aligned}$$

Thus, the resultant is

$$\mathbf{R} = 1088 \text{ N} \measuredangle\ 22.8^\circ \qquad \Leftarrow \textbf{Ans.}$$

Comparison of the two methods indicates clearly that the trigonometric method gives a more accurate solution. The degree of accuracy of the graphical solution can be improved, however, if a larger scale is used and more care is exercised when making the drawing. Much greater accuracy may be obtained if computer-aided drafting is used.

EXAMPLE 2–2

A 250-lb weight is lifted by pulling the two cords shown in Fig. E2–2(1). To lift the weight, the resultant of the two tensions $\mathbf{T}_1$ and $\mathbf{T}_2$ must be 250 lb acting vertically upward. Determine (a) the tension in each rope, knowing that $\theta = 40°$, and (b) the angle θ, for which the tension $\mathbf{T}_2$ is a minimum.

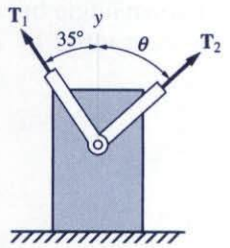


FIGURE E2–2(1)

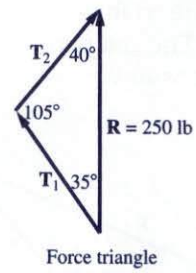


FIGURE E2–2(2)

Solution.

(a) Tensions for θ = 40°. A force triangle is drawn with $\mathbf{R} = 250$ lb vertically upward, and $\mathbf{T}_1$ and $\mathbf{T}_2$ in the directions shown in Fig. E2–2(2). By the law of sines, we write

$$\frac{T_1}{\sin 40°} = \frac{T_2}{\sin 35°} = \frac{250 \text{ lb}}{\sin 105°}$$

From which, we get

$$T_1 = 166 \text{ lb} \qquad T_2 = 148 \text{ lb} \qquad \Leftarrow \textbf{Ans.}$$

(b) Values of θ for Minimum $\mathbf{T}_2$. Refer to Fig. E2–2(3). Using the triangular rule, we first draw line *AB* to represent the known resultant. Then we draw line *A*1 from *A* along the known direction of $\mathbf{T}_1$. Several possible directions of $\mathbf{T}_2$ are represented by the lines marked *B*2. Among these lines, the shortest one representing $(\mathbf{T}_2)_{\min}$ is perpendicular to $\mathbf{T}_1$. Thus,

$$\theta = 90° - 35°$$
$$= 55° \qquad \Leftarrow \textbf{Ans.}$$

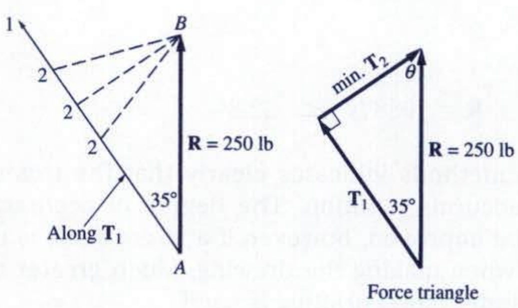


FIGURE E2–2(3)

EXAMPLE 2-3

Determine the resultant of the five forces shown in Fig. E2–3(1) by the graphical method.

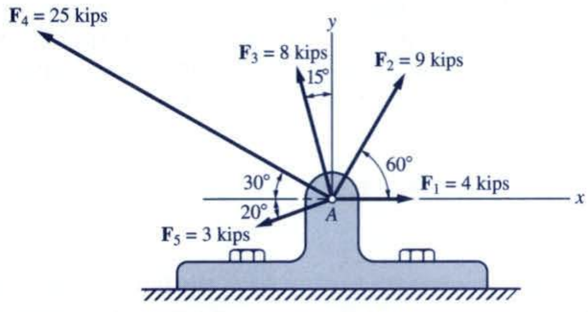


FIGURE E2–3(1)

Solution. Refer to Fig. E2–3(2). Starting from point *O*, draw *Oa, ab, bc, cd,* and *de,* representing forces $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$, and $\mathbf{F}_5$, respectively, in a head-to-tail fashion.

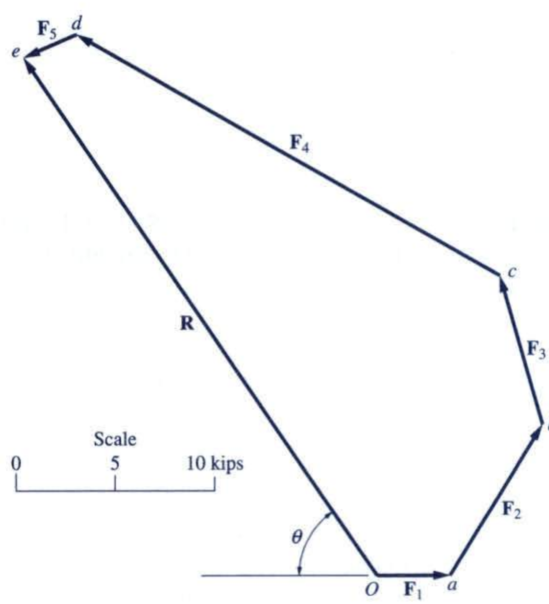


FIGURE E2–3(2)

The closing side of the polygon *Oe* is the desired resultant **R.** The magnitude and direction of **R** are measured to be

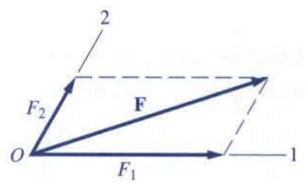
$$\mathbf{R} = 32.5 \text{ kips } \measuredangle\ 56° \qquad \Leftarrow \textbf{Ans.}$$

2–4 RECTANGULAR COMPONENTS

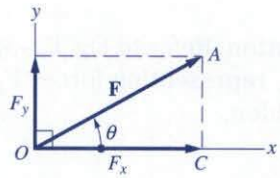
Any two or more forces whose resultant is equal to a force **F** are called the *components* of the force **F**. In Fig. 2–7a, $\mathbf{F}_1$ and $\mathbf{F}_2$ are the components of force **F** along the *O*1 and *O*2 directions. Two mutually perpendicular components are called the *rectangular components*. In Fig. 2–7b, $\mathbf{F}_x$ and $\mathbf{F}_y$ are the rectangular components of **F** in the *x* and *y* directions. We write

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

The *x* and *y* axes may be chosen in any two perpendicular directions. Usually the axes are chosen along horizontal and vertical directions.



(a) Components of a force along two arbitrary directions 1 and 2



(b) Rectangular components of a force along two perpendicular directions

FIGURE 2–7

Rectangular Components. If the magnitude *F* and the direction angle θ of a force are known, then, from the right triangle *OAC* in Fig. 2–7b, the rectangular components are

$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta \tag{2–2}$$

In Equation 2–2, the direction angle θ must be measured in the *standard position*; that is, it is measured from the positive *x* axis to the force vector **F**. Counterclockwise measurement is regarded as positive; clockwise measurement is regarded as negative, as indicated in Fig. 2–8. The components along the positive coordinate axes are positive and those along the negative coordinate axes are negative. If the direction angle is in the standard position, Equation 2–2 will yield the correct sign for the components.

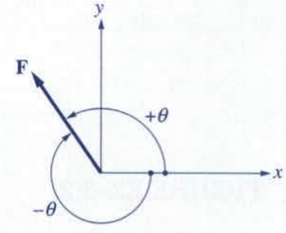


FIGURE 2–8

Magnitude and Direction. When the scalar components F_x and F_y of the force **F** are given, the magnitude of **F** may be determined from

$$F = \sqrt{F_x^2 + F_y^2} \tag{2–3}$$

and the reference angle α (a positive acute angle between the positive or negative x axis and the force vector) is

$$\alpha = \tan^{-1} \left| \frac{F_y}{F_x} \right| \tag{2–4}$$

Depending on which quadrant the force vector is in, the direction angle θ, in the standard position, is

$$\text{First quadrant: } \theta = \alpha \tag{2–5a}$$

$$\text{Second quadrant: } \theta = 180° - \alpha \tag{2–5b}$$

$$\text{Third quadrant: } \theta = 180° + \alpha \tag{2–5c}$$

$$\text{Fourth quadrant: } \theta = 360° - \alpha \quad \text{or} \quad \theta = -\alpha \tag{2–5d}$$

The quadrant that a force vector is in may be determined by using a sketch. For example, if both components are negative, a simple sketch will indicate that the force vector is in the third quadrant.

EXAMPLE 2–4

Resolve the 500-N force exerted on the hook in Fig. E2–4(1) into horizontal and vertical components.

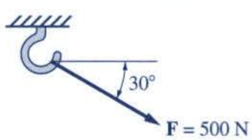


FIGURE E2–4(1)

Solution. The coordinate axes are chosen as shown in Fig. E2–4(2), where the positive x axis is horizontal to the right and the positive y axis is vertically upward. The reference angle is 30°. Since the force is in the fourth quadrant, angle θ in the standard position is

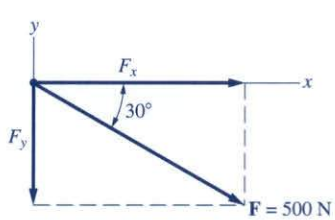


FIGURE E2–4(2)

$$\theta = 360^\circ - \alpha$$
$$= 360^\circ - 30^\circ$$
$$= 330^\circ \qquad (\text{or } -30^\circ)$$

Using Equation 2–2, we get

$$F_x = F\cos\theta$$
$$= (500 \text{ N}) \cos 330^\circ$$
$$= +433 \text{ N} \qquad \Leftarrow \textbf{Ans.}$$
$$F_y = F\sin\theta$$
$$= (500 \text{ N}) \sin 330^\circ$$
$$= -250 \text{ N} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–5

Refer to Fig. E2–5(1). The tension in guy wire *BC* is $T = 3.5$ kips. Resolve this force into the *x* and *y* components.

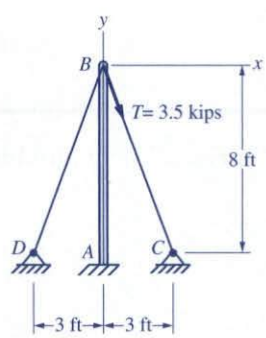


FIGURE E2–5(1)

Solution. The tension **T** is along wire *BC*, which has a slope of

$$\text{Horizontal} : \text{Vertical} = 3{:}8$$

as indicated in the slope triangle shown in Fig. E2–5(2). The hypotenuse in the slope triangle is

$$\sqrt{3^2 + 8^2} = \sqrt{73}$$

From the slope triangle, we get

$$\sin\alpha = \frac{8}{\sqrt{73}}$$

$$\cos\alpha = \frac{3}{\sqrt{73}}$$

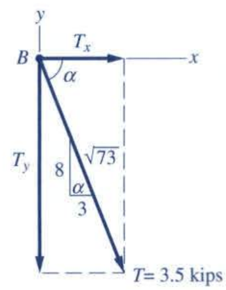


FIGURE E2–5(2)

Using Equation 2–2, we get

$$\begin{aligned} T_x &= +T\cos\alpha \\ &= +(3.5\text{ kips})\left(\frac{3}{\sqrt{73}}\right) \\ &= +1.23\text{ kips} \end{aligned}$$ ⇐ **Ans.**

$$\begin{aligned} T_y &= -T\sin\alpha \\ &= -(3.5\text{ kips})\left(\frac{8}{\sqrt{73}}\right) \\ &= -3.28\text{ kips} \end{aligned}$$ ⇐ **Ans.**

Since the angle α used is not in the standard position, we must assign a proper sign to each component by inspection.

EXAMPLE 2–6

Refer to Fig. E2–6(1). Resolve the 100-lb weight into components along the incline and normal to the incline.

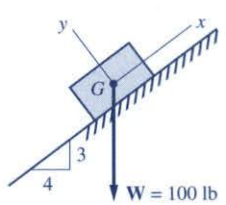


FIGURE E2–6(1)

Solution. The x and y axes are chosen along and normal to the incline, respectively. The components W_x and W_y are shown in Fig. E2–6(2). By inspection, we see that both components are negative. Thus,

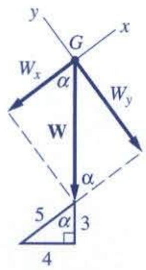


FIGURE E2–6(2)

$$W_x = -W\cos\alpha \qquad W_y = -W\sin\alpha$$

From the slope triangle (a 3-4-5 right triangle), we have

$$\cos\alpha = \frac{3}{5} \qquad \sin\alpha = \frac{4}{5}$$

Therefore,

$$W_x = -(100\text{ lb})\,\frac{3}{5}$$
$$= -60\text{ lb} \qquad \Leftarrow \textbf{Ans.}$$

$$W_y = -(100\text{ lb})\,\frac{4}{5}$$
$$= -80\text{ lb} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–7

Refer to Fig. E2–7. The rectangular components of a force **F** are given as $F_x = -30$ lb and $F_y = +40$ lb. Determine the magnitude and direction of the force.

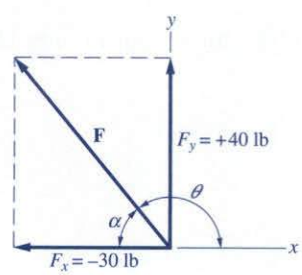


FIGURE E2–7

Solution. From Equation 2–3, the magnitude of the force is

$$F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$
$$= \sqrt{(-30\text{ lb})^2 + (40\text{ lb})^2}$$
$$= 50\text{ lb}$$

From Equation 2–4, the reference angle α is

$$\alpha = \tan^{-1}\left|\frac{F_y}{F_x}\right|$$
$$= \tan^{-1}\left|\frac{40}{-30}\right|$$
$$= 53.1^\circ$$

Since the force vector is in the second quadrant, from Equation 2–5b, the angle θ in the standard position is

$$\theta = 180^\circ - \alpha$$
$$= 180^\circ - 53.1^\circ$$
$$= 126.9^\circ$$

Thus,

$$\mathbf{F} = 50 \text{ lb} \;\angle\; 126.9^\circ \qquad \Leftarrow \textbf{Ans.}$$

2–5 RESULTANTS BY RECTANGULAR COMPONENTS

The resultant of any number of concurrent coplanar forces can be determined by using their rectangular components. Consider three coplanar forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at point O, as shown in Fig. 2–9. The resultant $\mathbf{R}$ of the three forces is

$$\boldsymbol{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3$$

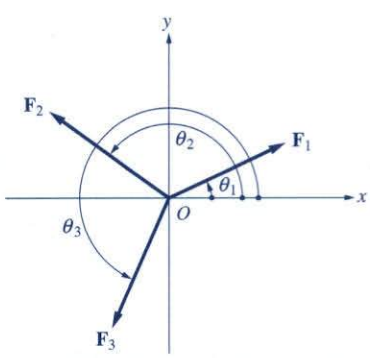


FIGURE 2–9

Each force is resolved into its rectangular components. All the x components are in the horizontal direction; hence, these components may be added algebraically. Similarly, all the y components may be added algebraically. In terms of the direction angle in the standard position, we write

$$R_x = (F_x)_1 + (F_x)_2 + (F_x)_3$$
$$= F_1\cos\theta_1 + F_2\cos\theta_2 + F_3\cos\theta_3$$
$$R_y = (F_y)_1 + (F_y)_2 + (F_y)_3$$
$$= F_1\sin\theta_1 + F_2\sin\theta_2 + F_3\sin\theta_3$$

The components of the resultant are the algebraic sums of the corresponding components of the forces. In general, for a system of coplanar forces, we write

$$R_x = \Sigma F_x = \Sigma F\cos\theta \qquad R_y = \Sigma F_y = \Sigma F\sin\theta \tag{2–6}$$

where the symbol Σ (Greek capital letter sigma) stands for "summation." The direction angle θ must be in the standard position. Once the scalar components of the resultant are obtained, the magnitude and direction of the resultant can be obtained from Equations 2–3, 2–4, and 2–5.

EXAMPLE 2–8

Determine the resultant of the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on the eye-bolt shown in Fig. E2–8(1).

FIGURE E2–8(1)

Solution. The direction angles of the forces in the standard position are indicated in Fig. E2–8(2). Using Equation 2–6, we write

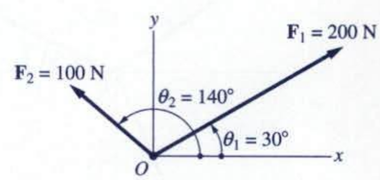


FIGURE E2–8(2)

$$R_x = F_1\cos\theta_1 + F_2\cos\theta_2$$
$$= (200\text{ N})\cos 30^\circ + (100\text{ N})\cos 140^\circ$$
$$= +96.6\text{ N}$$

$$R_y = F_1\sin\theta_1 + F_2\sin\theta_2$$
$$= (200\text{ N})\sin 30^\circ + (100\text{ N})\sin 140^\circ$$
$$= +164.3\text{ N}$$

Note that if the 50° angle from the y axis for $\mathbf{F}_2$ is used, we must pay close attention to the sign and the proper trigonometric functions to be used. We write

$$R_x = (200\text{ N})\cos 30° - (100\text{ N})\sin 50° = +96.6\text{ N}$$

$$R_y = (200\text{ N})\sin 30° + (100\text{ N})\cos 50° = +164.3\text{ N}$$

The magnitude of the resultant is

$$R = \sqrt{(96.6\text{ N})^2 + (164.3\text{ N})^2} = 190.6\text{ N}$$

Since both components are positive, the resultant is in the first quadrant. Thus, the direction angle is

$$\theta = \alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{164.3}{96.6}\right) = 59.5°$$

$$\mathbf{R} = 190.6\text{ N} \measuredangle\ 59.5° \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–9

Determine the resultant of the five forces in Example 2–3 by using rectangular components. The diagram is reproduced here as Fig. E2–9.

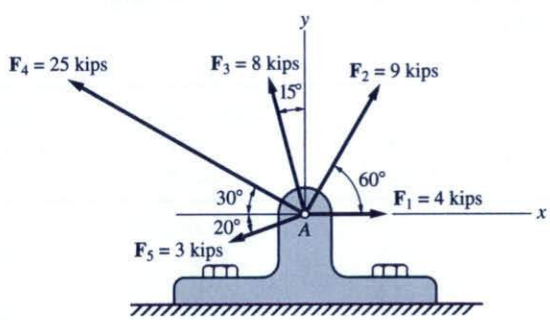


FIGURE E2–9

Solution. The solution is presented in two methods.

(a) By Equation. Using the given angles as they are, we find

$$R_x = \Sigma F_x = (F_x)_1 + (F_x)_2 + (F_x)_3 + (F_x)_4 + (F_x)_5$$
$$= 4 + (9)\cos 60° - (8)\sin 15° - (25)\cos 30° - (3)\cos 20°$$
$$= -18.04\text{ kips}$$

$$R_y = \Sigma F_x = (F_y)_1 + (F_y)_2 + (F_y)_3 + (F_y)_4 + (F_y)_5$$
$$= 0 + (9)\sin 60^\circ - (8)\cos 15^\circ - (25)\sin 30^\circ - (3)\sin 20^\circ$$
$$= +27.0 \text{ kips}$$

In the above equations, a plus or minus sign is assigned to each component by inspection. Components along positive directions of the axes are positive, and those along negative directions of the axes are negative. To decide which trigonometric function to use, just remember that to get a component opposite to the angle, the sine function of the angle must be used, and to get a component adjacent to the angle, the cosine function must be used. When making the calculation with a calculator, make sure that the calculator is in the degree mode.

With the x and y components determined, the magnitude of the resultant is

$$R = \sqrt{(18.04 \text{ kips})^2 + (27.0 \text{ kips})^2} = 32.5 \text{ kips}$$

With a negative x component and a positive y component, the resultant is in the second quadrant. Thus,

$$\alpha = \tan^{-1}\left|\frac{R_y}{R_x}\right| = \tan^{-1}\left|\frac{27.0}{-18.04}\right| = 56.3^\circ$$

$$\theta = 180^\circ - \alpha = 123.7^\circ$$

$$\mathbf{R} = 32.5 \text{ kips } \nwarrow 123.7^\circ$$ ⇐ **Ans.**

(b) By Tabulation. When the number of forces involved is greater than three, a solution in tabulated form is recommended. The x and y components of each force are listed in the table below. The angle θ of each force must be in the standard position.

Force	*Magnitude* *F (kips)*	*Angle* *θ (deg)*	*x component* F_x *(kips)* $= F\cos\theta$	*y component* F_y *(kips)* $= F\sin\theta$
F_1	4	0	4.00	0
F_2	9	60	4.50	7.79
F_3	8	105	−2.07	7.73
F_4	25	150	−21.65	12.50
F_5	3	200	−2.82	− 1.03
Σ			−18.04	+27.0

Thus,

$$R_x = -18.04 \text{ kips} \qquad R_x = -18.04 \text{ kips}$$

These are the same results obtained before.

2-6
MOMENT OF A FORCE

Effects of a Force. A force tends to move a body along its line of action. It also tends to rotate a body about an axis. For example, a pull on a door knob (Fig. 2–10) causes the door to rotate about the axis through the hinges. The ability of a force to cause a body to rotate is measured by a quantity called the *moment* of the force.

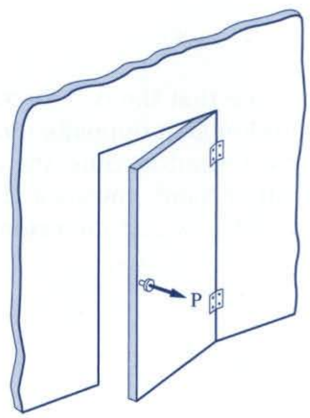


FIGURE 2–10

Consider a wrench used to tighten a bolt, as shown in Fig. 2–11. The rotating moment (also called the torque) produced by the applied force **F** depends not only on the magnitude of the force, but also on the perpendicular distance d from the center O of the bolt to the line of action of the force. In fact, the turning effect of the force is measured by the product of F and d.

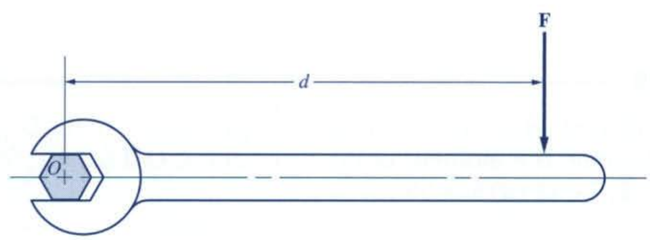


FIGURE 2–11

Definition of Moment. In the two-dimensional case, the moment M_O (Fig. 2–12) of a force F about a point O (called the *moment center*) is equal to the magnitude of the force **F** multiplied by the perpendicular distance d (called the *moment arm*) from O to the line of action of the force:

$$M_o = Fd \tag{2-7}$$

The units for moment are lb · ft or lb · in. in the U.S. customary units, and N · m or kN · m in the SI units.

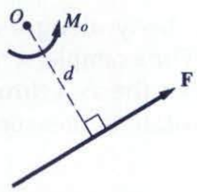


FIGURE 2–12

Direction of Moments. In Fig. 2–13, we see that the two forces **P** and **Q** cause the lever to rotate about the pivot point O in opposite directions. The force **P** causes a counterclockwise (c.c.w.) rotation; thus, the moment of **P** about O is c.c.w. The force **Q,** on the other hand, causes a clockwise (c.w.) rotation; thus, the moment of **Q** about O is c.w. It is important to distinguish whether a moment is c.w. or c.c.w.

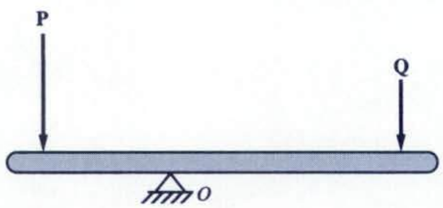


FIGURE 2–13

Summation of Moments. In the two-dimensional case, moments may be added algebraically. A proper sign must be assigned to the moment, depending on whether it is c.w. or c.c.w. In this book, unless stated otherwise, *a c.c.w. moment will be considered positive and a c.w. moment will be considered negative.*

EXAMPLE 2–10

A 500-N force is applied to the end of a lever pivoted at point O [see Fig. E2–10(1)]. Determine the moment of the force about O if (*a*) $\theta = 30°$, (*b*) $\theta = 120°$, (*c*) $\theta = 90°$, and (*d*) $\theta = 50°$.

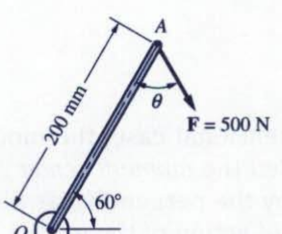


FIGURE E2–10(1)

Solution. The moment about O will be determined by definition as follows.

(a) $\theta = 30°$ ***for a Vertical Force.*** Refer to Fig. E2–10(2). The moment arm d is

$$d = (0.2 \text{ m}) \cos 60° = 0.1 \text{ m}$$

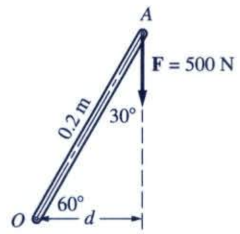


FIGURE E2–10(2)

The moment of the force about O is

$$M_O = Fd = -(500 \text{ N})(0.1 \text{ m}) = -50 \text{ N} \cdot \text{m}$$

Since the force tends to rotate the lever clockwise about O, the moment is clockwise.

$$M_O = 50 \text{ N} \cdot \text{m} \circlearrowright$$ ⇐ **Ans.**

(b) $\theta = 120°$ ***for a Horizontal Force.*** Refer to Fig. E2–10(3). The moment arm and the moment are

$$d = (0.2 \text{ m}) \sin 60° = 0.1732 \text{ m}$$

$$M_O = Fd = -(500 \text{ N})(0.1732 \text{ m}) = -86.6 \text{ N} \cdot \text{m} = 86.6 \text{ N} \cdot \text{m} \circlearrowright$$ ⇐ **Ans.**

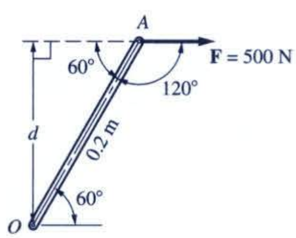


FIGURE E2–10(3)

(c) θ = 90° for a Perpendicular Force. See Fig. E2–10(4). In this case the entire length of the lever is the moment arm and the moment of the force about *O* has the maximum value.

$$\begin{aligned} M_O &= Fd \\ &= -(500\text{ N})(0.2\text{ m}) \\ &= -100\text{ N}\cdot\text{m} \\ &= 100\text{ N}\cdot\text{m} \circlearrowright \end{aligned}$$ ⇐ **Ans.**

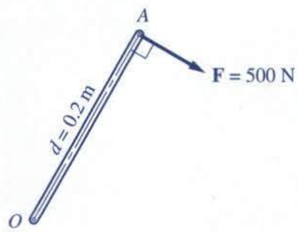


FIGURE E2–10(4)

(d) θ = 50° for an Inclined Force. Figure E2–10(5) represents a more general case. The moment arm can still be determined easily as:

$$\begin{aligned} d &= (0.2\text{ m})\sin 50^\circ \\ &= 0.1532\text{ m} \end{aligned}$$

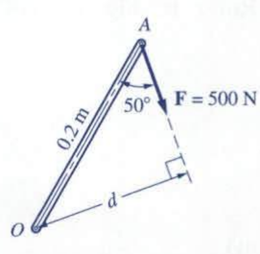


FIGURE E2–10(5)

The moment of the force about *O* is

$$\begin{aligned} M_O &= Fd \\ &= -(500\text{ N})(0.1532\text{ m}) \\ &= -76.6\text{ N}\cdot\text{m} \\ &= 76.6\text{ N}\cdot\text{m} \circlearrowright \end{aligned}$$ ⇐ **Ans.**

2-7 VARIGNON'S THEOREM

Varignon's theorem states that *the moment of a force about any point is equal to the sum of the moments produced by the components of the forces about the same point.* This theorem was established by the French mathematician Varignon (1654–1722). A formal proof of this theorem will not be given here. Intuitively, we see that any force can be resolved into components without altering its effect, so the sum of the moments of the components must be the same as the moment of the force itself.

Since the moment arm of a force is often hard or impossible to determine, Varignon's theorem is very useful for finding the moment of a force. In Fig. 2–14, if the coordinates of the point of application A of the force are (x_A, y_A), then the moment of the force about the origin O in terms of its rectangular components is

$$M_O = F_y x_A - F_x y_A \tag{2–8}$$

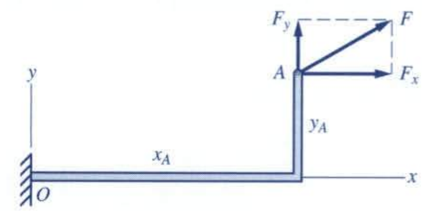


FIGURE 2–14

The *principle of transmissibility* is also helpful in calculating the moment of a force. Using this principle, the point of application of a force acting on a rigid body may be placed anywhere along its line of action. We see that the moment arm is clearly independent of the point of application of a force. Therefore, as long as the magnitude, the direction, and the line of action of a force are defined, the moment of a force about a given point may be determined by placing the force at *any* point along its line of action. For example, to find the moment of the force **F** (at A) in Fig. 2–15a about point O, we can resolve the force into rectangular components at B on the line of action of the force, as shown in Fig. 2–15b. Since the component F_x passes through the moment center O, it produces no moment about O. As a result, the moment of the force about O is simply

$$M_O = F_y x_B$$

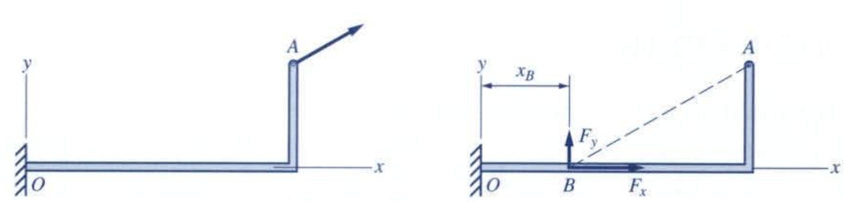


FIGURE 2–15

EXAMPLE 2–11

Determine the moment of the 100-lb force about point B in Fig. E2–11(1).

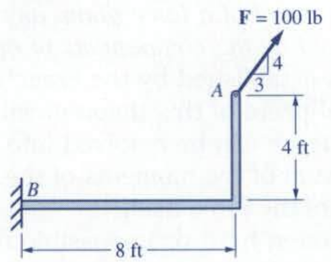


FIGURE E2–11(1)

Solution. The moment will be determined in three ways to illustrate different methods of solution.

(a) By Definition. From the geometry shown in Fig. E2–11(2), the moment arm d is

$$CD = AD/\tan\alpha$$
$$= (4\text{ ft}) / (4/3)$$
$$= 3\text{ ft}$$
$$BC = BD - CD$$
$$= 8\text{ ft} - 3\text{ ft}$$
$$= 5\text{ ft}$$
$$d = BC\sin\alpha$$
$$= (5\text{ ft})(4/5)$$
$$= 4\text{ ft}$$

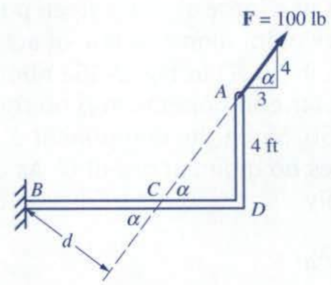


FIGURE E2–11(2)

By the definition of moment, we get

$$M_B = Fd$$
$$= +(100\text{ lb})(4\text{ ft})$$
$$= +400\text{ lb}\cdot\text{ft}$$

which is a counterclockwise moment. Thus,

$$M_B = 400 \text{ lb} \cdot \text{ft} \circlearrowleft \qquad \Leftarrow \textbf{Ans.}$$

(b) By Varignon's Theorem. Refer to Fig. E2–11(3). Resolve the force into horizontal and vertical components at A. The vertical component produces a counterclockwise moment and the horizontal component produces a clockwise moment. Thus,

$$M_B = (80 \text{ lb})(8 \text{ ft}) - (60 \text{ lb})(4 \text{ ft})$$
$$= +400 \text{ lb} \cdot \text{ft} \circlearrowleft$$

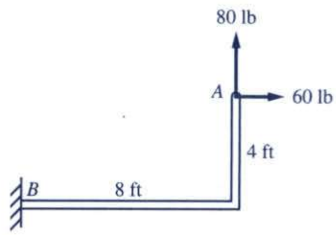


FIGURE E2–11(3)

(c) By the Principle of Transmissibility. Refer to Fig. E2–11(4). The force may be considered to act at C, where the force is resolved into horizontal and vertical components. The line of action of the horizontal component passes through point B and produces no moment about the point. Hence,

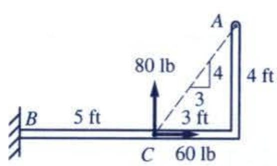


FIGURE E2–11(4)

$$M_B = +(80 \text{ lb})(5 \text{ ft})$$
$$= +400 \text{ lb} \cdot \text{ft} \circlearrowleft$$

EXAMPLE 2–12

Determine the maximum clockwise moment that can be produced by a 10-kN force exerted on the rectangular plate about the corner A in Fig. E2–12(1).

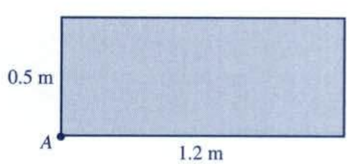


FIGURE E2–12(1)

Solution. To produce the maximum moment, the moment arm must be a maximum. See Fig. E2–12(2). This occurs when the point of application is at the opposite corner B and the line of action of the force is perpendicular to the diagonal AB. The moment arm is

$$d = AB$$
$$= \sqrt{(1.2 \text{ m})^2 + (0.5 \text{ m})^2}$$
$$= 1.3 \text{ m}$$

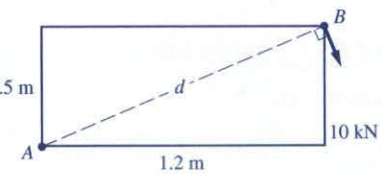


FIGURE E2–12(2)

The maximum moment is

$$M_A = Fd$$
$$= (10 \text{ kN})(1.3 \text{ m})$$
$$= 13 \text{ kN} \cdot \text{m} \circlearrowright \qquad \Leftarrow \textbf{Ans.}$$

2–8 COUPLE

Effect of a Couple. Two equal and opposite forces having parallel lines of action form a couple. Figure 2–16a shows a couple formed by two such forces. The sum of the two forces is zero. The sum of the moment of the two forces, however, is not zero. The effect of a couple acting on a rigid body, therefore, is to cause the rigid body to rotate about an axis perpendicular to the plane of the forces.

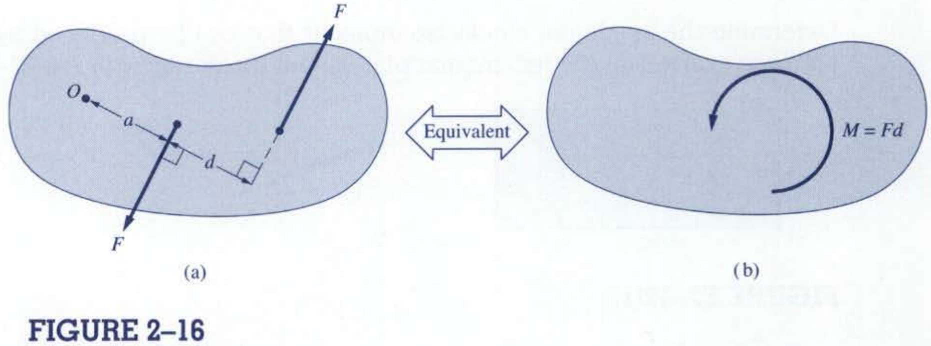


FIGURE 2–16

Moment of a Couple. Denoting the perpendicular distance between the two forces by d, the moment of a couple about an arbitrary point O is

$$M = F(a + d) - Fa = Fd \tag{2–9}$$

Since O is an arbitrary point, *the moment of a couple about any point is equal to the magnitude of the forces times the perpendicular distance between the forces.* Fig. 2–16b shows an alternative representation of a couple. A couple can be placed anywhere in the plane of the forces.

Equivalent Couples. Two couples acting on the same plane or parallel planes are equivalent if they have the same moment acting in the same direction; that is, two couples are equivalent if both the magnitude and the direction of their moments are equal.

Addition of Couples. The addition of two or more couples in a plane or parallel planes is the algebraic sum of their moments. Unless specified otherwise, we will treat a c.c.w moment as positive and a c.w. moment as negative.

EXAMPLE 2–13

Determine the moment of the couple applied to the torque wrench shown in Fig. E2–13 if the magnitude of **F** is 40 lb.

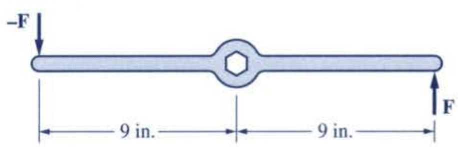


FIGURE E2–13

Solution. The perpendicular distance between the two forces is

$$\begin{aligned} d &= 9 \text{ in.} + 9 \text{ in.} \\ &= 18 \text{ in.} = 1.5 \text{ ft} \end{aligned}$$

By definition, the moment of the couple is

$$\begin{aligned} M &= Fd \\ &= +(40 \text{ lb})(1.5 \text{ ft}) \\ &= +60 \text{ lb} \cdot \text{ft} \end{aligned}$$

which produces a c.c.w. torque on the nut. Thus, the moment of the couple is

$$\text{M} = 60 \text{ lb} \cdot \text{ft} \circlearrowleft$$ ⇐ **Ans.**

EXAMPLE 2–14

Two couples act on the rectangular plate shown in Fig. E2–14(1).
(a) Determine the resultant moment of the two couples.
(b) Replace the couples by an equivalent couple formed by two smaller forces applied at the corners *A* and *C*.

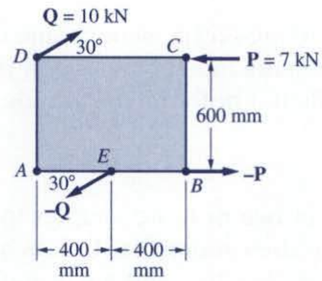


FIGURE E2–14(1)

Solution.

(a) Resultant Moment. Because the perpendicular distance between **Q** and **−Q** cannot be determined readily, the forces are resolved into horizontal and vertical components, as shown in Fig. E2–14(2). Now the two horizontal components form a c.w. couple and the two vertical components also form a c.w. couple. The resultant moment of the couples is the algebraic sum of the moment of each couple. Thus,

$$\begin{aligned} M &= +(7 \text{ kN})(0.6 \text{ m}) - (8.66 \text{ kN})(0.6 \text{ m}) - (5 \text{ kN})(0.4 \text{ m}) \\ &= -3.00 \text{ kN} \cdot \text{m} \\ &= 3.00 \text{ kN} \cdot \text{m} \circlearrowright \end{aligned}$$

⇐ **Ans.**

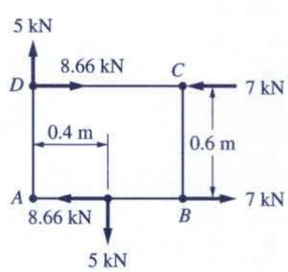


FIGURE E2–14(2)

(b) Equivalent Couple. The equivalent couple is formed by two equal and opposite forces **F** and **−F** applied at the corners *A* and *C* shown in Fig. E2–14(3). If the forces are to be the smallest, they must be perpendicular to the diagonal *AC*. Thus,

$$d = \sqrt{(0.8\text{ m})^2 + (0.6\text{ m})^2}$$
$$= 1.0\text{ m}$$
$$Fd = M$$
$$F = \frac{M}{d}$$
$$= \frac{3.00\text{ kN}\cdot\text{m}}{1.0\text{ m}}$$
$$= 3.00\text{ kN}$$

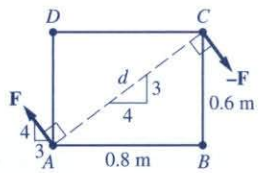


FIGURE E2–14(3)

The equivalent couple is formed by two forces

$\mathbf{F} = 3.00$ kN (slope 4 vertical : 3 horizontal) at A and $-\mathbf{F}$ at C ⇐ **Ans.**

2–9 REPLACING A FORCE WITH A FORCE-COUPLE SYSTEM

Two systems of forces are said to be equivalent if they produce the same mechanical effect on a rigid body. The mechanical effect of any system of forces on a rigid body is characterized entirely by the resultant force and the resultant moment of the system. Hence, we define *equivalent force systems* as follows.

Equivalent Force Systems. Systems of forces are said to be *equivalent* if they have *the same resultant force and the same resultant moment about the same point.*

Force-Couple System. Consider a force **F** acting on a rigid body at point A (Fig. 2–17a). Suppose that it is necessary to move the force to another point B. From the principle of transmissibility, we know that we can move the force to any point along its line of action, but we cannot move the force to *any* point B. We may, however, add two equal and opposite forces **F** and −**F** at point B (Fig. 2–17b) without altering the mechanical effect of the original force. Now we have a force **F** at B, and a couple formed by force **F** at A and force −**F** at B. The moment of the couple is

$$M = Fd \qquad (2\text{–}10)$$

where d is the perpendicular distance from point B to the line of action of the given force at A. We see that the moment M is simply the moment of the original force at A about point B. Since the moment of the couple about any point is the same, it may be placed anywhere in the plane. For convenience, however, the force and the couple are usually shown to act at the same point B (Fig. 2–17c) and we refer to this combination as a *force-couple system.* Thus, *any given force may be moved to another point without changing its mechanical effect, provided that an appropriate couple is added. The couple has a moment produced by the given force about the point where the force is to be located.*

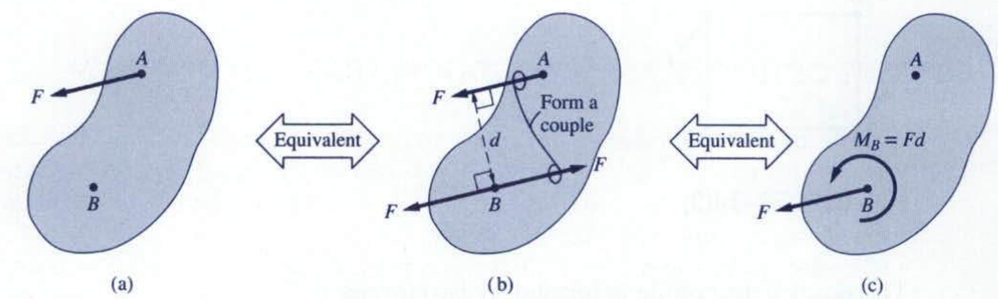


FIGURE 2–17

EXAMPLE 2–15

Replace the 2-kN force **F** shown in Fig. E2–15(1) with the force-couple system at point B.

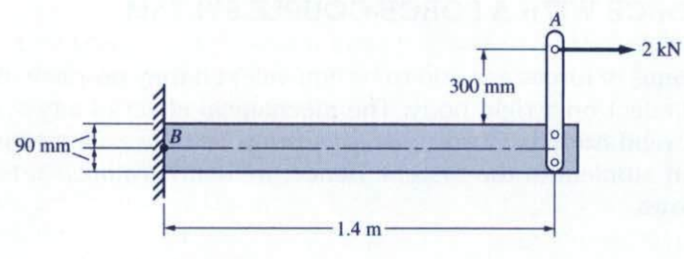


FIGURE E2–15(1)

Solution. Refer to Fig. E2–15(2). The force **F** can be moved from point A to point B, provided that a suitable couple is introduced. The moment of the couple is equal to the moment of the force at A about point B. We have

$$M_B = -\ (2\ \text{kN})(0.3\ \text{m} + 0.09\ \text{m})$$
$$= -0.78\ \text{kN}\cdot\text{m} \ \circlearrowright \qquad \Leftarrow \textbf{Ans.}$$

Thus, the single force at A is now replaced by an equivalent force-couple system at point B, as shown in Fig. E2–15(2).

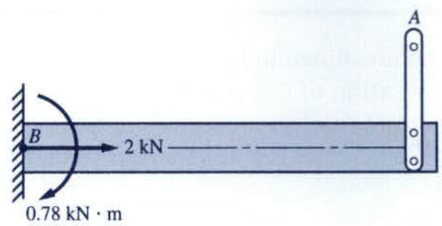


FIGURE E2–15(2)

2–10 RESULTANT OF A NONCONCURRENT COPLANAR FORCE SYSTEM

In a concurrent coplanar force system, the line of action of the resultant passes through the common point. In a nonconcurrent coplanar force system, there is no point of concurrency, so the location of the line of action of the resultant is not immediately known.

The magnitude and direction of the resultant can be calculated by using the rectangular components of the forces, similar to the method used for the concurrent coplanar force system. First, we choose convenient x and y coordinate axes and then resolve each force into rectangular components. *The components of the resultant are the algebraic sums of the corresponding components of all the forces in the system.* We write

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2–11}$$

From these components, the magnitude and direction of the resultant can be determined. Now the location of the line of action of the resultant can be determined by the requirement of the moments. If two force systems are equivalent, the resultant moments of the two systems about an arbitrary point must be equal. Consider the given force system $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_4$ acting on the beam shown in Fig. 2–18. The resultant $\mathbf{R}$ of the given force system is assumed to act through point C at distance $\bar{x}$ to the right of point A. The moment of $\mathbf{R}$ about A must be the same as the sum of the moments of the given forces about A. Note that R_x passes through A, so it produces no moment about A. We write

$$R_y \bar{x} = \Sigma M_A$$

From which $\bar{x}$ can be solved.

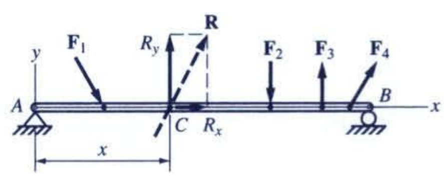


FIGURE 2–18

EXAMPLE 2–16

Three forces are applied to the beam shown in Fig. E2–16(1). Find the resultant of the three forces and the location of the resultant.

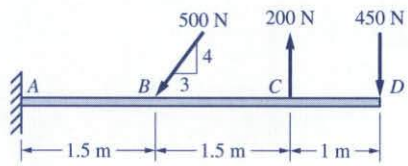


FIGURE E2–16(1)

Solution. The x and y coordinate axes are selected as shown in Fig. E2–16(2). The 500-N force is resolved into x and y components.

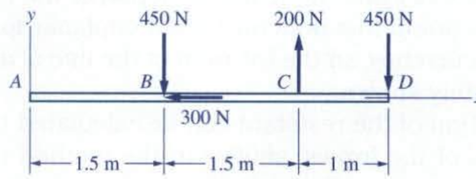


FIGURE E2–16(2)

Resultant. Summing up the corresponding components, we get

$$R_x = \Sigma R_x$$
$$= -300 \text{ N} \leftarrow$$
$$R_y = \Sigma R_y$$
$$= -400 \text{ N} + 200 \text{ N} - 450 \text{ N}$$
$$= -650 \text{ N} \downarrow$$

From the components of the resultant, we see that it is a vector in the third quadrant. The magnitude and direction of the resultant are

$$R = \sqrt{(-300 \text{ N})^2 + (-650 \text{ N})^2} = 716 \text{ N}$$
$$\alpha = \tan^{-1} \left| \frac{650}{300} \right|$$
$$= 65.2^\circ$$
$$\theta = 180^\circ + 65.2^\circ$$
$$= 245.2^\circ$$
$$\mathbf{R} = 716 \text{ N} \;\angle\; 245.2^\circ \qquad \Leftarrow \textbf{Ans.}$$

Location of Resultant. To determine the location of the resultant, we need to find the sum of the moments of the forces about A.

$$\Sigma M_A = -(400\text{ N})(1.5\text{ m}) + (200\text{ N})(3.0\text{ m}) - (450\text{ N})(4.0\text{ m})$$
$$= -1800\text{ N}\cdot\text{m} \circlearrowright$$

Since R_y is downward, the resultant **R** must be to the right of point A to produce a clockwise moment about A. Equating the moment of **R** about point A to the moment found above, we write

$$(\circlearrowright +): (650\text{ N})\,\bar{x} = 1800\text{ N}\cdot\text{m}$$

From which we get

$$\bar{x} = +2.77\text{ m} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–17

For the given force system acting on the bracket shown in Fig. E2–17(1), find the resultant and the point of intersection of the resultant along AD.

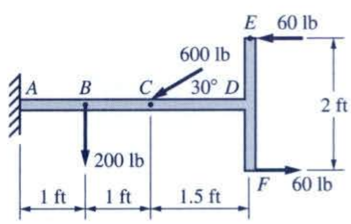


FIGURE E2–17(1)

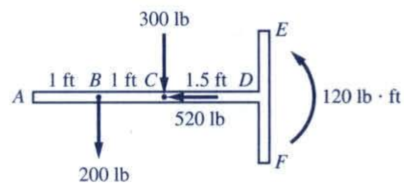


FIGURE E2–17(2)

Solution. Refer to Fig. E2–17(2). The two 60-lb forces acting in opposite directions form a couple. The moment of the couple is

$$M = (60\text{ lb})(2\text{ ft})$$
$$= 120\text{ lb}\cdot\text{ft} \circlearrowright$$

The couple is represented by its moment, and the 600-lb force is replaced by its rectangular components.

Resultant. The components of the resultant are

$$R_x = \Sigma F_x$$
$$= -520\text{ lb} \longleftarrow$$
$$R_y = \Sigma F_y$$
$$= -200\text{ lb} - 300\text{ lb}$$
$$= -500\text{ lb} \downarrow$$

The resultant is a vector in the third quadrant. The magnitude and direction of the resultant are

$$R = \sqrt{(-520\text{ lb})^2 + (-500\text{ lb})^2}$$
$$= 721\text{ lb}$$
$$\alpha = \tan^{-1}\left|\frac{500}{520}\right| = 43.9°$$
$$\theta = 180° + 43.9°$$
$$= 223.9°$$
$$\mathbf{R} = 721\text{ lb} \;\angle\; 223.9° \qquad \Leftarrow \textbf{Ans.}$$

Location of Resultant. To determine the location of the resultant, we need to sum up the moment of the forces about A.

$$\Sigma M_A = -(200\text{ lb})(1.0\text{ ft}) - (300\text{ lb})(2.0\text{ ft}) + 120\text{ lb}\cdot\text{ft}$$
$$= -680\text{ lb}\cdot\text{ft} \circlearrowright$$

Note that the moment of a couple is independent of the moment center, so the moment of the couple about A remains 120 lb · ft. Since R_y is downward, the resultant **R** must be to the right of point A to produce a clockwise moment about A. Equating the moment of **R** about A to the moment found above, we write

$$(\circlearrowright +):\ (500\text{ lb})\,\bar{x} = 680\text{ lb}\cdot\text{ft}$$

From which we get

$$\bar{x} = +1.36\text{ ft} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–18

Determine the resultant of the three forces and a couple acting on the plate shown in Fig. E2–18(1). Find the point of intersection of the resultant along the x and y axes.

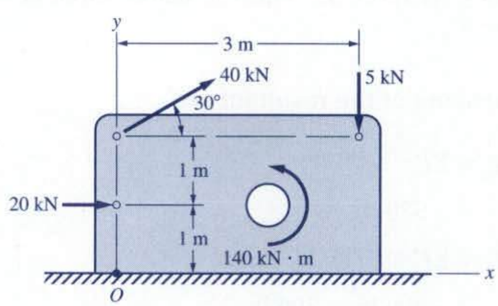


FIGURE E2–18(1)

Solution. The 40-kN force is replaced by its rectangular components and the couple is moved to point O, as shown in Fig. E2–18(2).

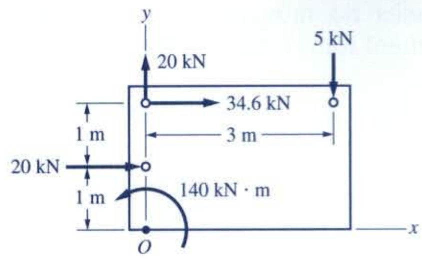


FIGURE E2–18(2)

Resultant. The components of the resultant are

$$\begin{aligned} R_x &= \Sigma F_x \\ &= 20 \text{ kN} + 34.6 \text{ kN} \\ &= 54.6 \text{ kN} \longrightarrow \\ R_y &= \Sigma F_y \\ &= 20 \text{ kN} - 5 \text{ kN} \\ &= 15.0 \text{ kN} \uparrow \end{aligned}$$

Thus, the resultant is a vector in the first quadrant. The magnitude and direction of the resultant are

$$\begin{aligned} R &= \sqrt{(54.6 \text{ kN})^2 + (15 \text{ kN})^2} \\ &= 56.6 \text{ kN} \\ \theta = \alpha &= \tan^{-1}\frac{15}{54.6} \\ &= 15.4^\circ \end{aligned}$$

$$\mathbf{R} = 56.6 \text{ kN} \measuredangle\ 15.4^\circ \qquad \Leftarrow \textbf{Ans.}$$

Location of Resultant. To determine the location of the resultant, we need to find the sum of the moments of the forces about O.

$$\begin{aligned} \Sigma M_O &= -(20 \text{ kN})(1.0 \text{ m}) - (34.6 \text{ kN})(2.0 \text{ m}) - (5.0 \text{ kN})(3.0 \text{ m}) + 140 \text{ kN} \cdot \text{m} \\ &= +35.7 \text{ kN} \cdot \text{m} \circlearrowleft \end{aligned}$$

The resultant **R** must be to the right and below point O to produce a counterclockwise moment about O.

At the x intercept, R_x produces no moment about O. Equating the moment of R_y about O to the moment found above, we write

$$(\circlearrowleft +): (15.0 \text{ kN})\,\bar{x} = 35.7 \text{ kN} \cdot \text{m}$$

From which we get

$$\bar{x} = 2.38 \text{ m} \qquad \Leftarrow \textbf{Ans.}$$

At the y intercept, R_y produces no moment about O. Equating the moment of R_x about O to the moment found above, we write

$$(\circlearrowleft +): (54.6 \text{ kN}) \, |\bar{y}| = 35.7 \text{ kN} \cdot \text{m}$$

From which we get

$$|\bar{y}| = 0.654 \text{ m} \qquad \Leftarrow \textbf{Ans.}$$

The location of the resultant is indicated in Fig. E2–18(3).

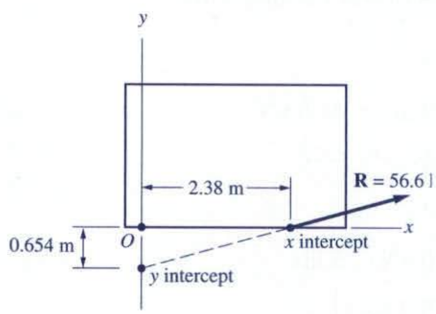


FIGURE E2–18(3)

2–11 RESULTANT OF DISTRIBUTED LINE LOADS

Distributed Load. A *distributed load* occurs whenever the load applied to a body is not concentrated at a point. A distributed load could be exerted along a line, over an area, or throughout an entire solid body. This section deals only with distributed line loads. Examples of such loading include the weight of a beam or the load from the floor system that the beam supports, and loads caused by wind or liquid pressure.

Load Intensity. A distributed load along a line is characterized by a load intensity expressed as force per unit length. For example, a load intensity of 1000 lb/ft, or 1 kip/ft, means that a load of 1000 lb, or 1 kip, is distributed over 1 ft length. In S.I. units, the load intensity is in N/m or kN/m.

Uniform Load. A distributed load with a constant intensity is called a *uniform load.* A uniform load may be represented by a loading diagram in the shape of a rectangular block, as shown in Fig. 2–19a. The uniform intensity w is represented by the height of the block, and the length of distribution b is represented by the width of the block. The weight of a beam of uniform cross-section is an example of a uniform load.

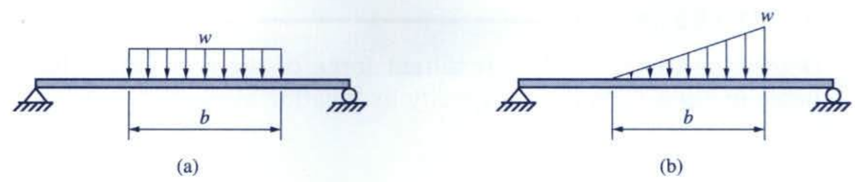


FIGURE 2–19

Triangular Load. A *triangular load* is a distributed load whose intensity varies linearly from zero to a maximum intensity *w*. A triangular load may be represented by a loading diagram in the shape of a triangle, as shown in Fig. 2–19b. Liquid pressure can be represented by a triangular load.

Equivalent Concentrated Force. For the purpose of determining the resultant of a force system, each distributed load may be replaced by its equivalent concentrated force. It will be established later in Section 7–5 that *a distributed line load may be replaced by an equivalent concentrated force having a magnitude equal to the area of the loading diagram and a line of action passing through the centroid of the loading diagram.* The equivalent concentrated forces of a uniform load and a triangular load are shown in Fig. 2–20a and b, respectively.

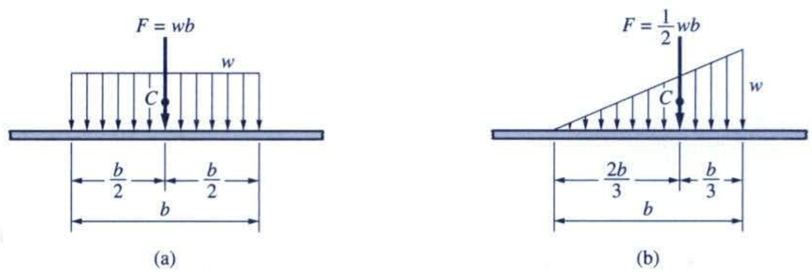


FIGURE 2–20

Trapezoidal Load. A load diagram in the shape of a trapezoid can be treated as a uniform load plus a triangular load, as shown in Fig. 2–21a. The general case of distributed load shown in Fig. 2–21b will be treated later in Section 7–5.

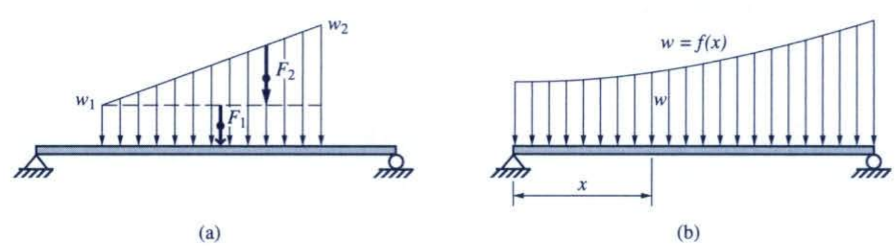


FIGURE 2–21

EXAMPLE 2–19

Determine the equivalent resultant force of the distributed loads on the beam in Fig. E2–19(1), and specify its location along the beam.

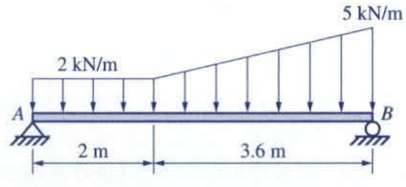


FIGURE E2–19(1)

Solution. The loading diagram is divided into a rectangle and a triangle, as shown in Fig. E2–19(2). The rectangle represents a uniformly distributed load of a constant intensity of 2 kN/m. The triangle represents a load with an intensity varying linearly from 0 to 3 kN/m.

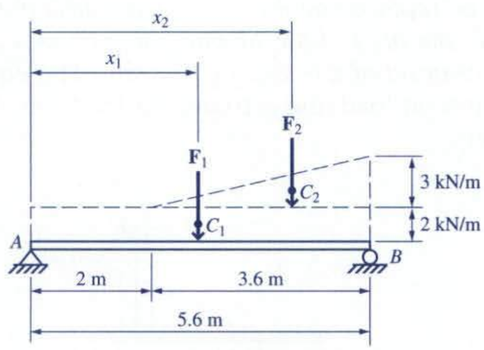


FIGURE E2–19(2)

Resultant. The distributed loads may be replaced by their equivalent concentrated forces of magnitudes equal to their associated areas. We have

$$F_1 = (2 \text{ kN/m})(5.6 \text{ m}) = 11.2 \text{ kN}$$

$$F_2 = \frac{1}{2}(3 \text{ kN/m})(3.6 \text{ m}) = 5.4 \text{ kN}$$

The line of action of each equivalent concentrated force passes through the *centroid* of the associated area of its loading diagram. The distances from A to the lines of action of the forces are

$$x_1 = \frac{1}{2}(5.6 \text{ m}) = 2.8 \text{ m}$$

$$x_2 = 2 \text{ m} + \frac{2}{3}(3.6 \text{ m}) = 4.4 \text{ m}$$

The magnitude of the resultant is

$$(\downarrow +): R = F_1 + F_2$$
$$= 11.2\text{ kN} + 5.4\text{ kN}$$
$$\mathbf{R} = 16.6\text{ kN} \downarrow \qquad \Leftarrow \textbf{Ans.}$$

Location of Resultant. Refer to Fig. E2–19(3). The distance $\bar{x}$ from A to the line of action of **R** may be obtained by equating the moment of **R** about A to the sum of the moments of $\mathbf{F}_1$ and $\mathbf{F}_2$ about A. We write

$$(\circlearrowright +): (16.6\text{ kN})\bar{x} = (11.2\text{ kN})(2.8\text{ m}) + (5.4\text{ kN})(4.4\text{ m})$$

From which we get

$$\bar{x} = 3.32\text{ m} \qquad \Leftarrow \textbf{Ans.}$$

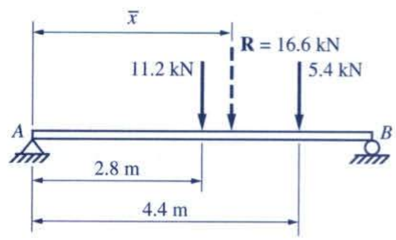


FIGURE E2–19(3)

EXAMPLE 2–20

Determine the equivalent resultant force of the loads acting on the beam shown in Fig. E2–20(1), and specify the location of the resultant along the beam.

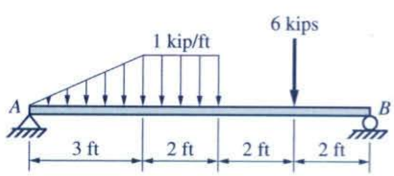


FIGURE E2–20(1)

Solution. Note that this combined loading consists of a concentrated load, a uniform load, and a triangular load.

Resultant. The two distributed loads may be replaced by their equivalent concentrated forces of magnitudes:

$$F_1 = \frac{1}{2}(1 \text{ kip/ft})(3 \text{ ft})$$
$$= 1.5 \text{ kips}$$
$$F_2 = (1 \text{ kip/ft})(2 \text{ ft})$$
$$= 2 \text{ kips}$$

Each of the equivalent concentrated forces acts vertically downward through the *centroid* of the associated area of its loading diagram, as shown in Fig. E2–20(2). The magnitude of the resultant is

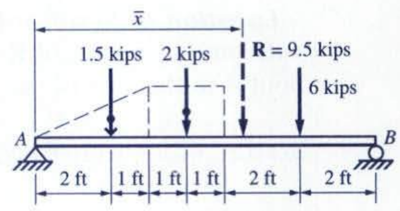


FIGURE E2–20(2)

$$(\downarrow +): R = \Sigma F_y = 1.5 \text{ kips} + 2 \text{ kips} + 6 \text{ kips}$$
$$= 9.5 \text{ kips}$$
$$\mathbf{R} = 9.5 \text{ kips} \downarrow \qquad \Leftarrow \textbf{Ans.}$$

Location of Resultant. The distance $\bar{x}$ from point A to the line of action of **R** may be obtained by equating the moment of **R** about A to the sum of the moment of the forces about A. We write

$$(\circlearrowright +): (9.5 \text{ kips})\bar{x} = (1.5 \text{ kips})(2 \text{ ft}) + (2 \text{ kips})(4 \text{ ft}) + (6 \text{ kips})(7 \text{ ft})$$

From which we get

$$\bar{x} = 5.58 \text{ ft} \qquad \Leftarrow \textbf{Ans.}$$

EXAMPLE 2–21

The bracket ABC is subjected to a uniform load and a trapezoidal load, as shown in Fig. E2–21(1). Replace the loads with an equivalent resultant force and specify its location along BC measured from the fixed end C.

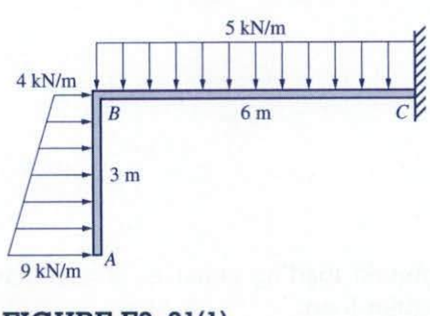


FIGURE E2–21(1)

Solution. The trapezoidal load is divided into a uniform load and a triangular load, as shown in Fig. E2–21(2).

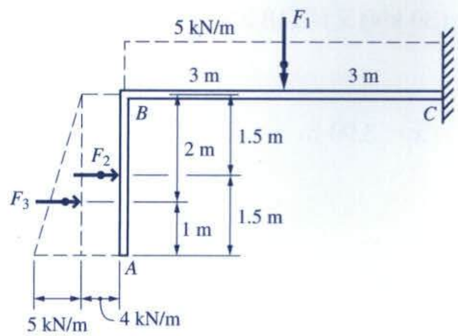


FIGURE E2–21(2)

Resultant. The equivalent concentrated forces of the distributed loads are

$$F_1 = (5 \text{ kN/m})(6 \text{ m}) = 30 \text{ kN}$$
$$F_2 = (4 \text{ kN/m})(3 \text{ m}) = 12 \text{ kN}$$
$$F_3 = \frac{1}{3}(5 \text{ kN/m})(3 \text{ m}) = 5 \text{ kN}$$

Each equivalent concentrated force passes through the centroid of the associated area of its loading diagram, as shown in Fig. E2–21(2). Refer to Fig. E2–21(3). The components of the resultant force are

$$R_x = \Sigma F_x = 12 \text{ kN} + 5 \text{ kN} = 17 \text{ kN} \longrightarrow$$
$$R_y = \Sigma F_y = -30 \text{ kN} \downarrow$$
$$\mathbf{R} = 34.5 \text{ kN} \; 60.5^\circ \qquad \Leftarrow \textbf{Ans.}$$

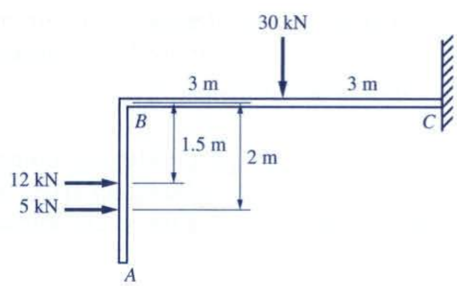


FIGURE E2–21(3)

Location of Resultant. Refer to Fig. E2–21(3). The resultant moment of the forces about *C* is

$$(\circlearrowleft +)\text{: } \Sigma M_C = (30 \text{ kN})(3\text{m}) + (12 \text{ kN})(1.5 \text{ m}) + (5 \text{ kN})(2 \text{ m})$$
$$= 118 \text{ kN} \cdot \text{m} \circlearrowleft$$

The single resultant force acting at point *D* along *BC* in Fig. E2–21(4) must produce the same c.c.w. moment about *C* as calculated above. We write

$$(\circlearrowleft +): (30 \text{ kN})\,\bar{x} = 118 \text{ kN} \cdot \text{m}$$

From which we get

$$\bar{x} = 3.93 \text{ m} \qquad \Leftarrow \textbf{Ans.}$$

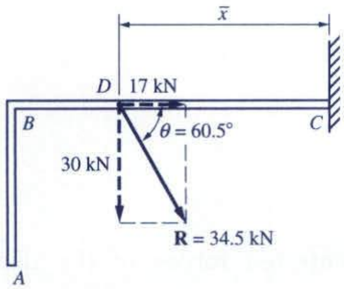


FIGURE E2–21(4)

2–12 SUMMARY

Resultant. A system of coplanar forces acting on a rigid body may be replaced by a single equivalent force called its *resultant,* which will produce the same mechanical effect to the rigid body as the given system. The determination of the resultant of a given coplanar force system is the major topic of this chapter.

Parallelogram Law. The resultant of two concurrent coplanar forces can be determined graphically by using the *parallelogram law* or the *triangular rule.* The resultant of three or more concurrent coplanar forces can be determined graphically by using the *polygon rule.*

Rectangular Components. Two mutually perpendicular components whose resultant is equal to a force are called the *rectangular components* of the force. If the magnitude and direction angle of a force are known, then the rectangular components are

$$F_x = F\cos\theta \qquad F_y = F\sin\theta \tag{2–2}$$

where the direction angle θ must be measured in the *standard position.* When the rectangular components of the force are given, the magnitude and direction of the force may be determined from

$$F = \sqrt{F_x^2 + F_y^2} \tag{2–3}$$

and the reference angle α is

$$\alpha = \tan^{-1}\left|\frac{F_y}{F_x}\right| \tag{2–4}$$

The direction angle θ in the standard position can then be determined.

Component Method. The resultant of any number of concurrent coplanar forces may be obtained by using the *component method.* First, each force is resolved into the x and y components. Then the components of the resultant can be determined from

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \tag{2–6}$$

From the two components R_x and R_y, the magnitude and direction of the resultant **R** can be determined. The line of action of the resultant **R** must pass through the common point of the given concurrent force system.

Moment. The *moment of a force* about a *moment center* is defined as the product of the magnitude of the force and the *moment arm,* which is the perpendicular distance from the *moment center* to the line of action of the force. Sometimes it is more convenient to determine the moment of a force by using its components. *Varignon's theorem* states that the moment of a force about a point is equal to the sum of the moments of the components of the force about the same point. By using the *principle of transmissibility,* it is more convenient to resolve the force into rectangular components at a point along its line of action, where only one component produces a moment about the given point.

Couple. A *couple* is produced by two equal, opposite, and noncollinear forces. The moment of a couple is equal to the magnitude of the force multiplied by the perpendicular distance between the two forces. A couple is characterized by its moment, which is independent of the moment center. Two couples on the same plane or parallel planes are equivalent if they have moments of the same magnitude and direction.

Force-Couple System. A force acting on a rigid body may be replaced by an equivalent *force-couple system* at an arbitrary point O consisting of the force applied at O and a couple having a moment equal to the moment about O of the given force at the original location.

Resultant of a Nonconcurrent Coplanar Force System. A nonconcurrent coplanar force system can be replaced by a single resultant force. The components of the resultant force may be determined the same way as those of the concurrent coplanar force system. From the two components, the magnitude and direction of the resultant can be determined. The location of the line of action of the resultant must be such that its moment about O will be equal to the sum of the moments of the given forces about O.

Resultant of Distributed Line Loads. For the purpose of determining the resultant of a force system, each distributed force is replaced by its equivalent concentrated force as follows:

For *uniform load:* $\mathbf{F} = wb$ acting through the midpoint of length b

For *triangular load:* $\mathbf{F} = \frac{1}{2}wb$ acting through the centroid of the triangle

PROBLEMS

Section 2–3 Resultant of Concurrent Forces

2–1 Determine graphically the magnitude and direction of the resultant of the two forces shown in Fig. P2–1 using (*a*) the parallelogram law and (*b*) the triangle rule.

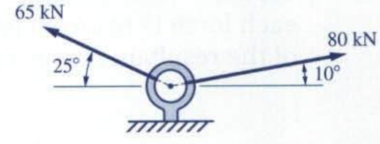


FIGURE P2–1

2–2 Solve Problem 2–1 by the trigonometric method.

2–3 Determine the magnitude and direction of the resultant of two forces acting on the eye hook shown in Fig. P2–3 by (*a*) the graphical method and (*b*) the trigonometric method.

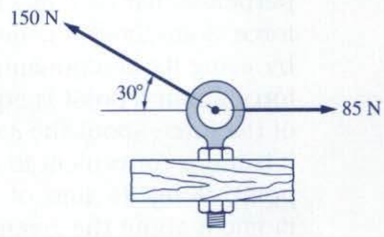


FIGURE P2–3

2–4 Determine the magnitude and direction of the resultant of the two forces acting on the bracket shown in Fig. P2–4.

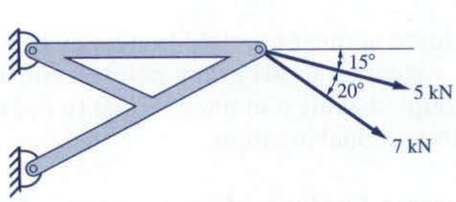


FIGURE P2–4

2–5 Determine the magnitude of the force **P** so that the resultant of the two forces acting on the block shown in Fig. P2–5 is vertical.

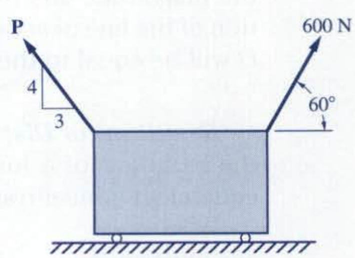


FIGURE P2–5

2–6 A trolley is acted on by two forces as shown in Fig. P2–6. If **P** = 3.5 kN, find the value of angle α so that the resultant of the forces is in the vertical direction.

2–7 If $\alpha = 40^\circ$ for the force **P** acting on the trolley shown in Fig. P2–6, find the magnitude of force **P** so that the resultant of the two forces is vertical.

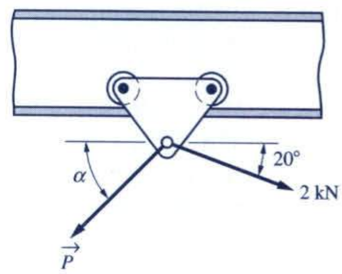


FIGURE P2–6

2–8 If the resultant of the two forces **P** and **Q** acting on the ring in Fig. P2–8 is a vertical force equal to 45 kN, find the magnitude and direction of force **Q.**

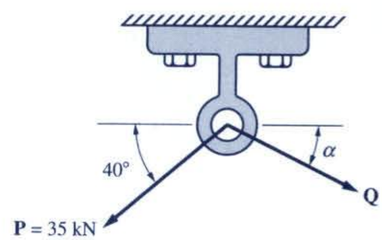


FIGURE P2–8

2–9 A barge is pulled by two tugboats as shown in Fig. P2–9. The tension in cable *AC* is 1000 lb. Determine the tension in cable *AB* if the resultant of the cable tensions is along the *x* axis.

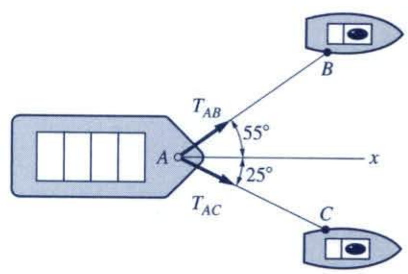


FIGURE P2–9

2–10 The resultant of cable tension **T** and a 5-kN weight must act along the axis of boom *AB* of the derrick shown in Fig. P2–10. Determine (*a*) the magnitude of tension **T** if $\theta = 30^\circ$, and (*b*) the value of angle θ for which the tension **T** is a minimum.

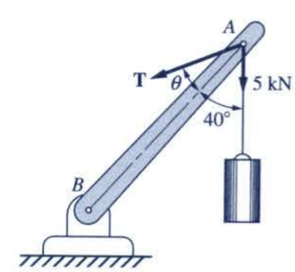


FIGURE P2–10

2–11 To have compressive soil pressure exist over the entire base of a gravity dam, the resultant of the forces acting on the dam above the base must pass through the middle third of the base. For the gravity dam shown in Fig. P2–11, the force **W** represents the weight of the dam for a one-foot section. The total water pressure acting on the one-foot section is represented by the horizontal force **P.** Determine the resultant of **W** and **P.** Is it within the middle third of the base?

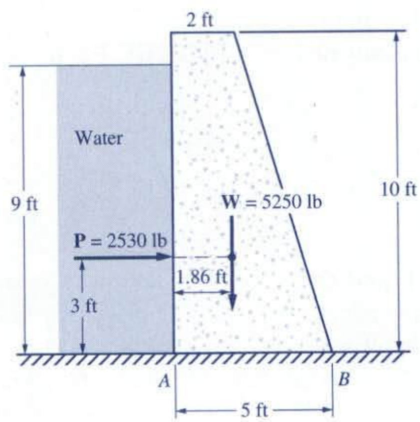


FIGURE P2–11

2–12 Determine by the graphical method the resultant of the three forces acting on the eye hook shown in Fig. P2–12.

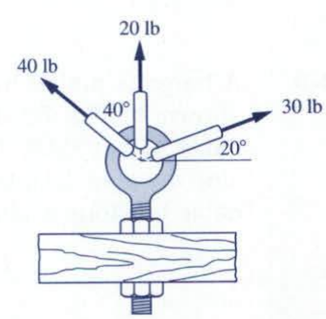


FIGURE P2–12

2–13 Determine by the graphical method the magnitude and direction of the resultant of the four forces shown in Fig. P2–13.

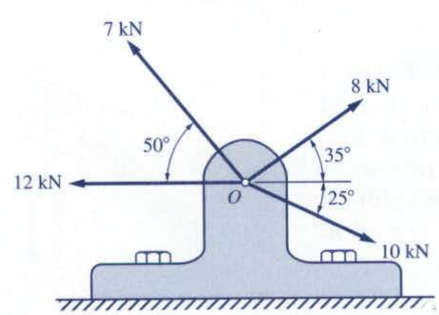


FIGURE P2–13

Section 2–4 Rectangular Components

2–14 to **2–17** Determine the x and y components of the forces shown in Figs. P2–14 to P2–17.

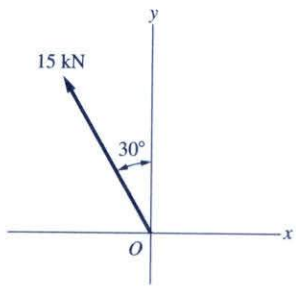


FIGURE P2–14

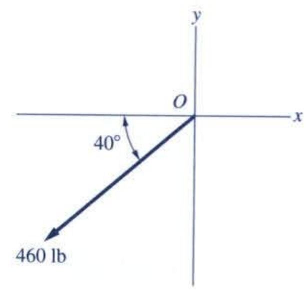


FIGURE P2–15

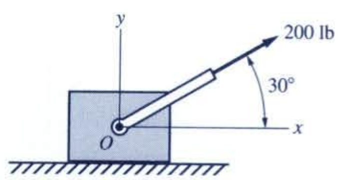


FIGURE P2–16

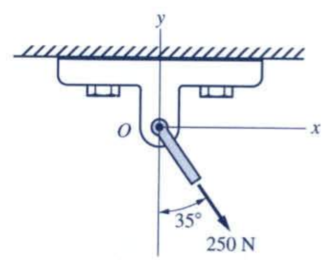


FIGURE P2–17

2–18 Refer to Fig. P2–18. Prove that the x and y components of a force acting in a direction indicated by the ratio $h{:}v$ are

$$F_x = \frac{h}{\sqrt{h^2 + v^2}} F \qquad F_y = \frac{v}{\sqrt{h^2 + v^2}} F$$

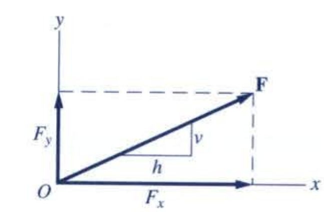


FIGURE P2–18

2–19 and **2–20** Find the x and y components of the forces shown in Figs. P2–19 and P2–20 by using the formulas in Problem 2–18.

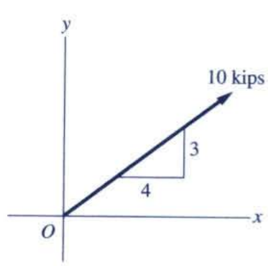


FIGURE P2–19

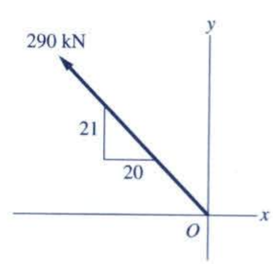


FIGURE P2–20

2–21 and **2–22** Find the x and y components of the forces **P** and **Q** shown in Figs. P2–21 and P2–22.

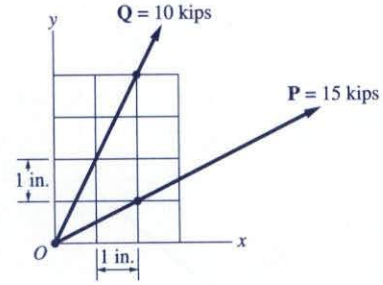


FIGURE P2–21

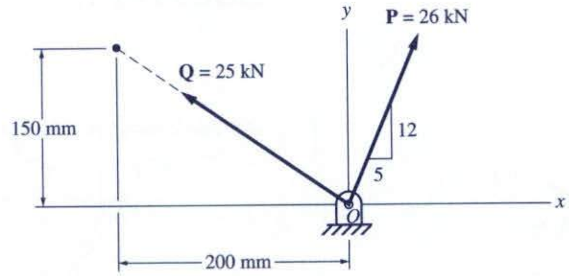
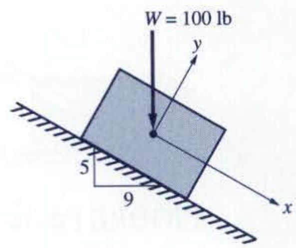


FIGURE P2–22



2–23 Find the x and y components of the weight of the block shown in Fig. P2–23.

FIGURE P2–23

Section 2–5 Resultants by Rectangular Components

2–24 to **2–27** Determine the magnitude and direction of the resultant of the force systems shown in Figs. P2–24 to P2–27.

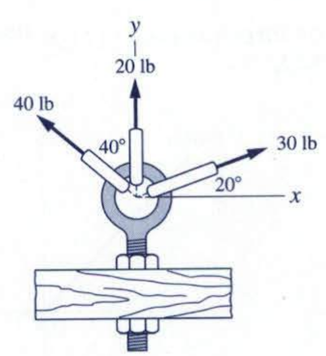


FIGURE P2–24

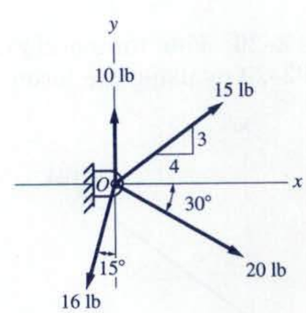


FIGURE P2–25

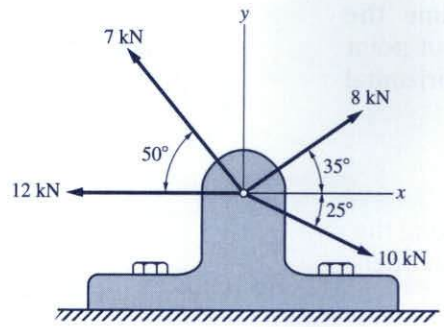


FIGURE P2–26

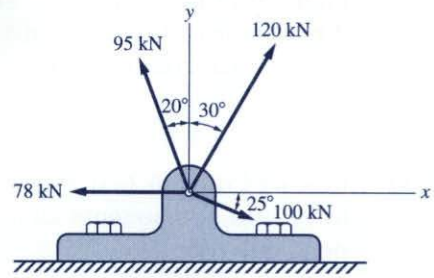


FIGURE P2–27

2–28 A collar that slides freely on a horizontal rod is subjected to the three forces shown in Fig. P2–28. Determine the angle θ for which the resultant of the three forces is vertical.

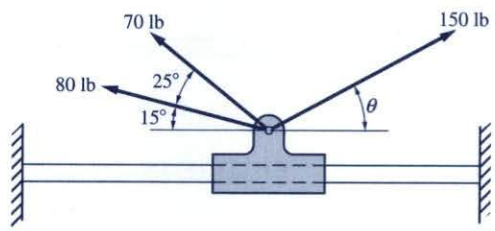


FIGURE P2–28

Section 2–6 Moment of a Force
Section 2–7 Varignon's Theorem

2–29 Refer to Fig. P2–29. Determine the moment of the 10-kN force about point O.
HW

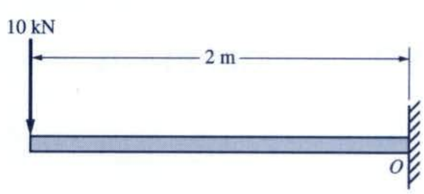


FIGURE P2–29

2–30 Refer to Fig. P2–30. Determine the moment of the 10-kip force shown about A by (*a*) using the definition directly and (*b*) resolving the force into horizontal and vertical components.

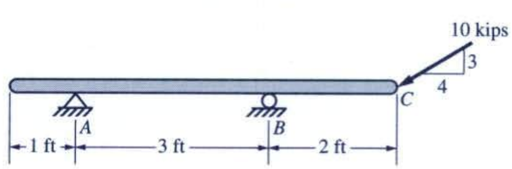


FIGURE P2–30

2–31 Refer to Fig. P2–31. Determine the moment of the 50-lb force about point *A* by resolving the force into horizontal and vertical components.

2–32 Rework Problem 2–31 by resolving the force into components along and perpendicular to *AB*.

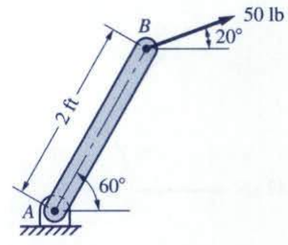


FIGURE P2–31

2–33 Refer to Fig. P2–33. Determine the moment of the 2000-N force shown about *A* by (*a*) using the definition directly, (*b*) resolving the force into horizontal and vertical components at *C*, and (*c*) resolving the force into components at *D*.

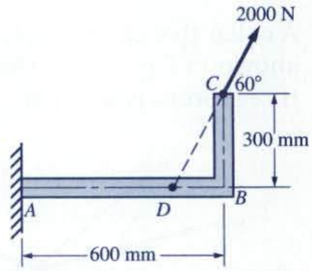


FIGURE P2–33

HW **2–34** Refer to Fig. P2–34. Determine the moment of the 200-N force about point *B* if α is 60°.

2–35 Refer to Fig. P2–34. Determine (*a*) angle α for which the moment of the 200-N force **F** about point *B* is a maximum and (*b*) the maximum moment.

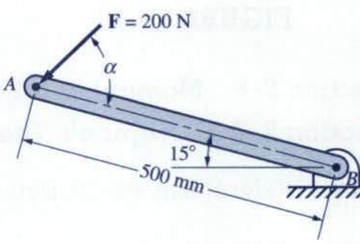


FIGURE P2–34

2–36 Refer to Fig. P2–36. Determine the total moment of the two forces about point *O*.

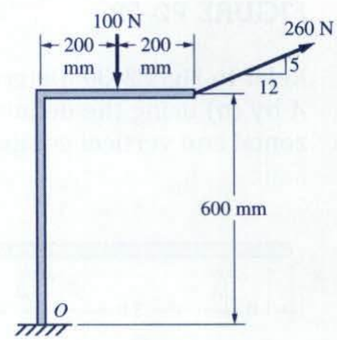


FIGURE P2–36

2–37 Determine the total moment of the three forces about point D in Fig. P2–37.

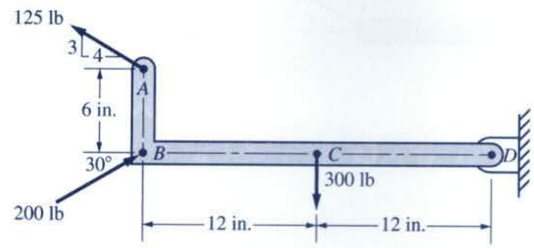


FIGURE P2–37

2–38 Refer to Fig. P2–38. Determine (*a*) the moment of the 400-lb force about point *O*, (*b*) the magnitude and direction of a vertical force applied at *B* that will produce the same moment about *O*, and (*c*) the smallest force applied at *C* that will produce the same moment about *O*.

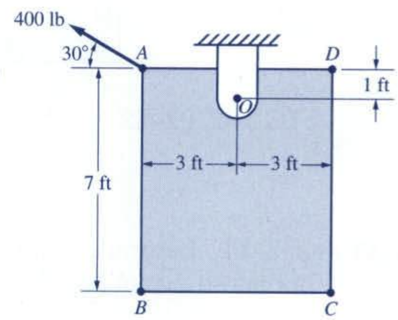


FIGURE P2–38

2–39 Refer to Fig. P2–39. Determine the moment of the 50-kN force about (*a*) the center *O* and (*b*) point *B*.

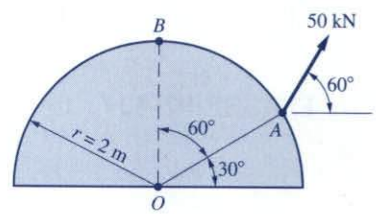


FIGURE P2–39

Section 2–8 Couple

2–40 to **2–42** Determine the moment of the couple acting on the bodies shown in Figs. P2–40 to P2–42.

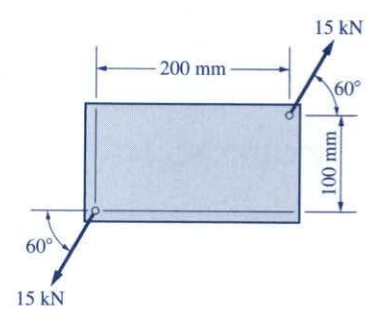


FIGURE P2–40

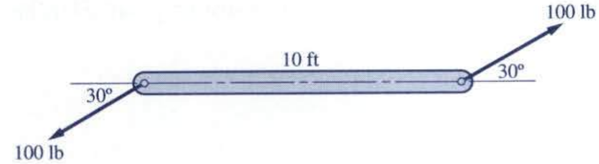


FIGURE P2–41

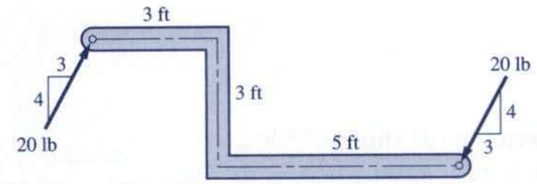


FIGURE P2–42

2–43 and **2–44** Determine the resultant moment of the couples acting on the bodies shown in Figs. P2–43 and P2–44.

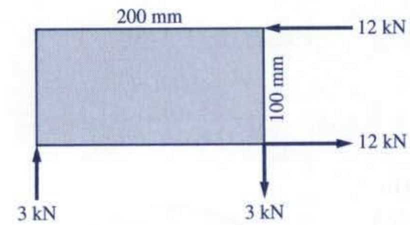


FIGURE P2–43

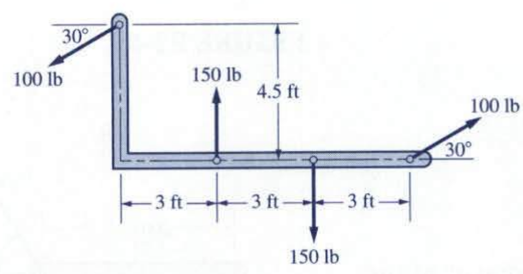


FIGURE P2–44

2–45 Two couples are applied to the 3-ft by 4-ft rectangular plate shown in Fig. P2–45. Prove that the total moment of the couples is zero by (*a*) adding the moment of the couples and (*b*) showing that the resultant of the two forces acting at the corner *A* is equal to, opposite to, and collinear with the resultant of the two forces acting at the corner *B*.

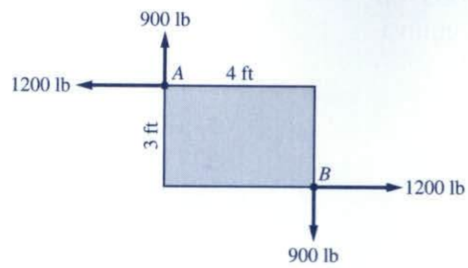


FIGURE P2–45

2–46 The angle bracket shown in Fig. P2–46 is subjected to the two 5-kN forces applied at points *A* and *B*. Replace these forces with an equivalent system consisting of the 7-kN force applied at point *C* and a second force applied at point *D*. Determine the magnitude and the direction of the second force at *D* and the distance *CD*.

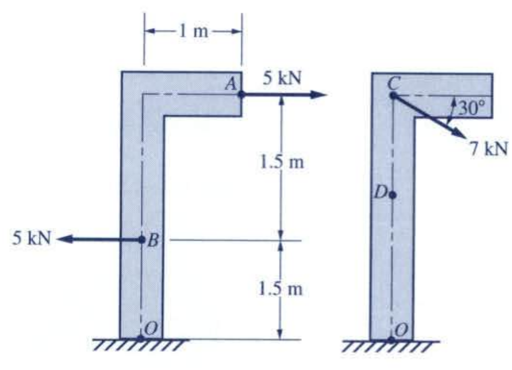


FIGURE P2–46

2–47 The plate in Fig. P2–47 is subjected to a system of forces that form three couples as shown. Determine the resultant moment of the couples.

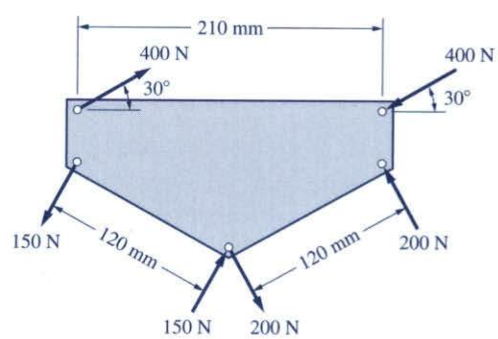


FIGURE P2–47

Section 2–9 Replacing a Force with a Force-Couple System

2–48 Replace the 5-kN horizontal force on the lever in Fig. P2–48 with an equivalent force-couple system at *O*.

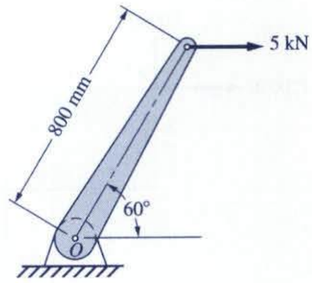


FIGURE P2–48

2–49 Replace the 10-kip force acting on the post in Fig. P2–49 with an equivalent force-couple system at *C*.

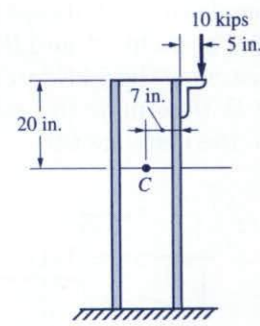


FIGURE P2–49

2–50 Replace the 2-kip force in Fig. P2–50 with an equivalent force-couple system at *B*.

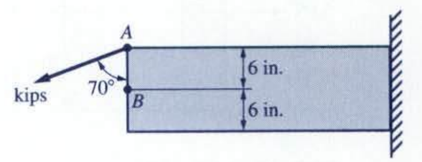


FIGURE P2–50

2–51 Replace the 600-lb force acting on the connection in Fig. P2–51 with an equivalent force-couple system at the center of rivet *B*.

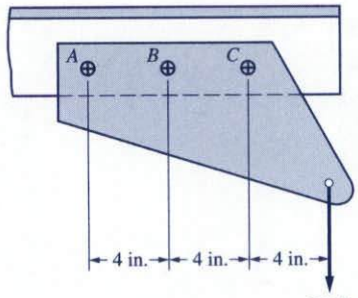


FIGURE P2–51

HW **2–52** Refer to Fig. P2–52. Replace the 500-N force applied to the bracket at A with an equivalent force-couple system at B.

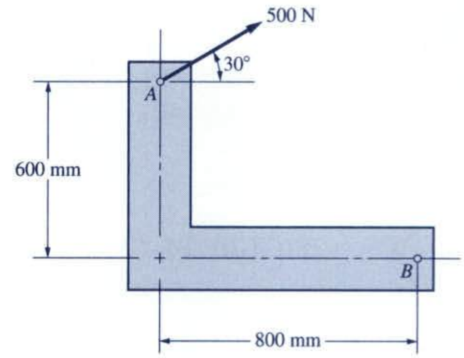


FIGURE P2–52

2–53 Replace the 20-lb force exerted on the wrench in Fig. P2–53 with an equivalent force-couple at O.

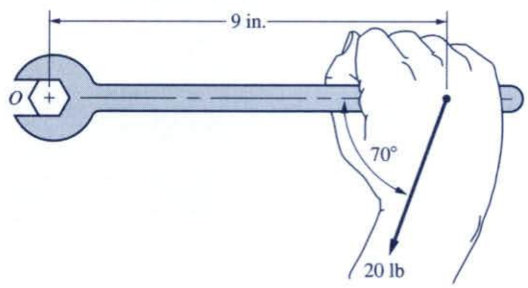


FIGURE P2–53

2–54 In designing the lift hook shown in Fig. P2–54, we must replace the 5-ton force with a force-couple at point B of section a–a. If the moment of the couple is 2500 lb-ft, determine the distance d.

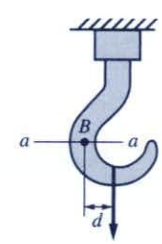


FIGURE P2–54

Section 2–10 Resultant of a Nonconcurrent Coplanar Force System

2–55 Replace the force and couple in Fig. P2–55 with a single force applied at a point on the diameter *AB*. Determine the distance from the center *O* to the point of application of the single force.

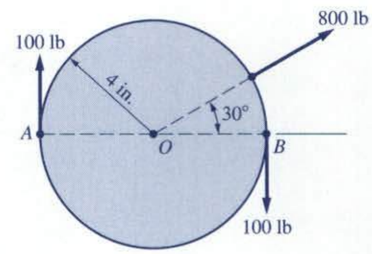


FIGURE P2–55

2–56 Replace the force and couple in Fig. P2–56 with a single force acting at a point along *AB*.

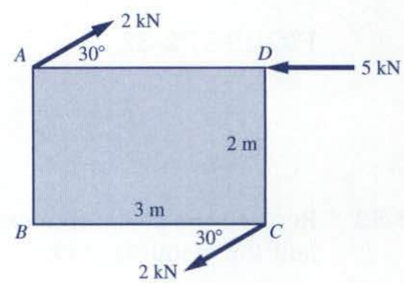


FIGURE P2–56

2–57 Replace the force and couple in Fig. P2–57 with a single force applied at a point along *AB*.

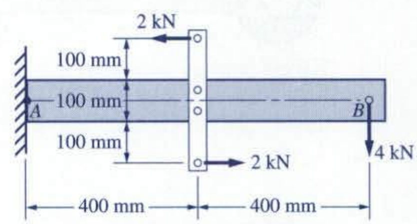


FIGURE P2–57

2–58 If the force-couple exerted on the beam in Fig. P2–58 can be replaced with an equivalent single force at *B*, find the magnitude of force **F**.

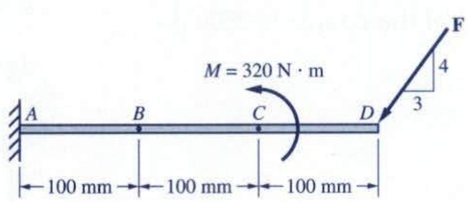


FIGURE P2–58

2–59 If the 350-N force and the couple M in Fig. P2–59 can be replaced with an equivalent single force at the corner C, determine the moment of the couple.

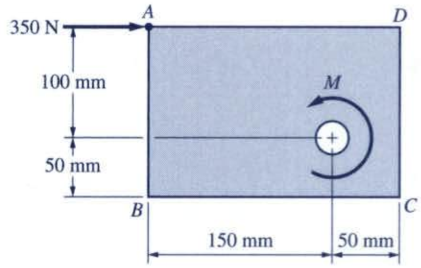


FIGURE P2–59

2–60 Refer to Fig. P2–60. Reduce the forces acting on the beam to a single resultant force and determine its point of application along AB.

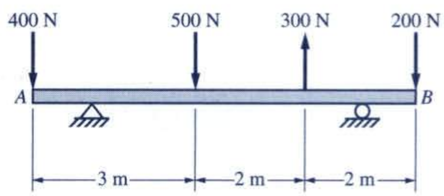


FIGURE P2–60

2–61 Refer to Fig. P2–61. Determine the height of the point above the base B through which the resultant of the three forces passes.

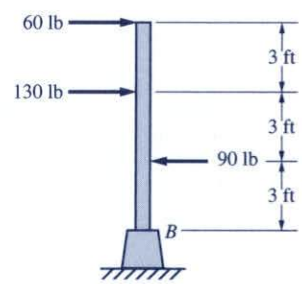


FIGURE P2–61

2–62 The trolley shown in Fig. P2–62 can be moved freely along the rail. Determine the location of the resultant of the two forces from point A when (a) $a = 2$ m and (b) $a = 3$ m.

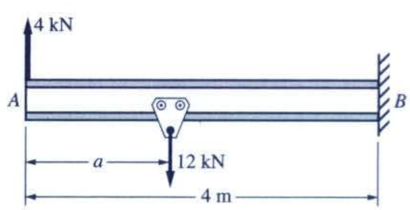


FIGURE P2–62

2–63 Explain why the resultant of the three forces acting on the beam in Fig. P2–63 always passes through point *A* for any value of force **F**.

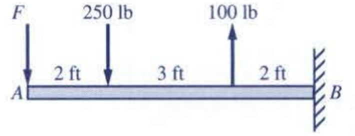


FIGURE P2–63

2–64 Find the magnitude, direction, and location of the resultant of the three forces acting on the beam in Fig. P2–64.

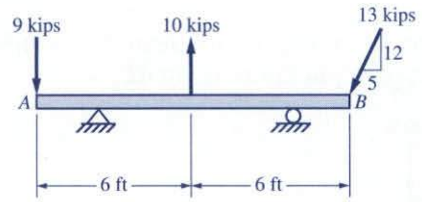


FIGURE P2–64

2–65 Refer to Fig. P2–65. Determine the magnitude of the vertical force **F** if the resultant of the three forces acting on the crank passes through the bearing *O*.

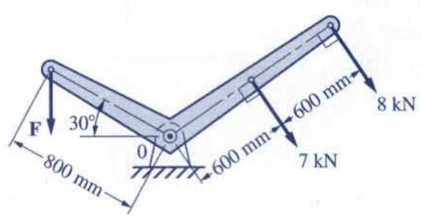


FIGURE P2–65

2–66 Determine the resultant of the four forces on the truss in Fig. P2–66 and its location along *AB*.

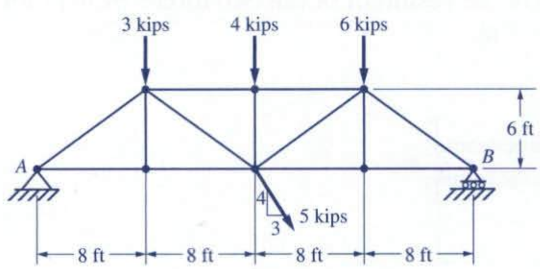


FIGURE P2–66

2–67 Replace the three forces acting on the frame in Fig. P2–67 with an equivalent force-couple system at *A*. Find the location of the single resultant above *A*.

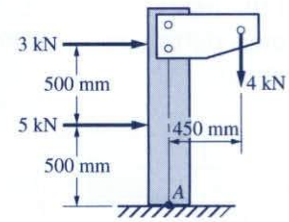


FIGURE P2–67

2–68 Determine the magnitude, direction, and location of the resultant of the two forces and a couple acting on the beam in Fig. P2–68.

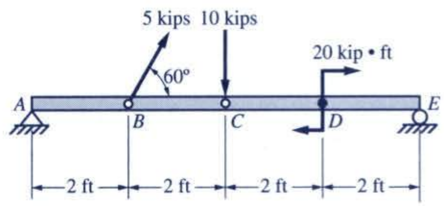


FIGURE P2–68

2–69 For the angle bracket in Fig. P2–69 subjected to the system of forces and couple shown, determine the resultant force and the points of intersection of its line of action with the *x* and *y* axes.

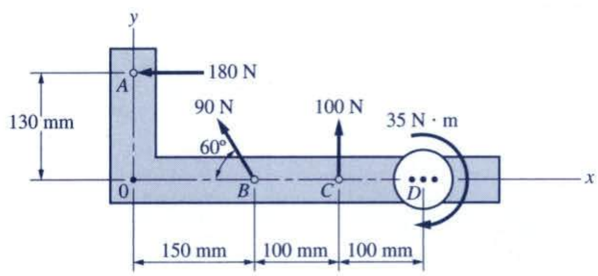


FIGURE P2–69

2–70 For the bracket in Fig. P2–70 subjected to the system of forces and couples shown, determine the single resultant and the point of intersection of its line of action with line *BC*.

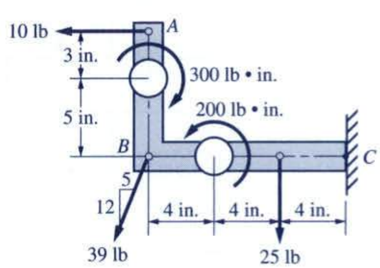


FIGURE P2–70

2–71 For the bracket in Fig. P2–71 subjected to the system of forces and couples shown, determine the point along line AC where the single resultant passes through.

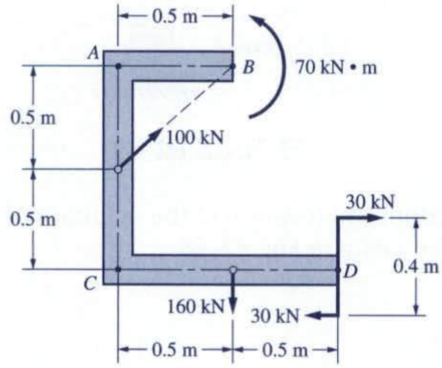


FIGURE P2–71

2–72 Reduce the force system acting on the bracket in Fig. P2–72 to the simplest form.

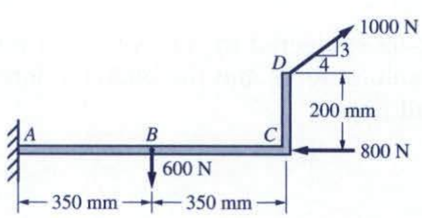


FIGURE P2–72

2–73 To have compressive soil pressure exerted over the entire base of a retaining wall, the resultant of the forces acting on the wall above the base must pass through the middle third of the base. For the retaining wall shown in Fig. P2–73, the vertical forces represent the weight of the concrete wall and footing, and the weight of the earth above the footing for a one-foot section of the wall. The horizontal force represents the total earth pressure acting on a one-foot section. Determine the location where the resultant passes through the base. Is it within the middle third of the base?

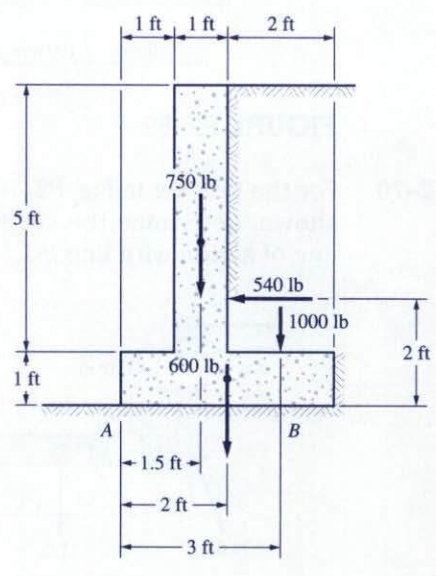


FIGURE P2–73

2–74 To have compressive soil pressure exerted over the entire base of a gravity dam, the resultant of the forces acting on the dam above the base must pass through the middle third of the base. For the gravity dam shown in Fig. P2–74, the weight of two parts of the dam for a one-meter section is shown. The total water pressure acting on the one-meter section is represented by the horizontal force. Determine the location where the resultant passes through the base. Is it within the middle third of the base?

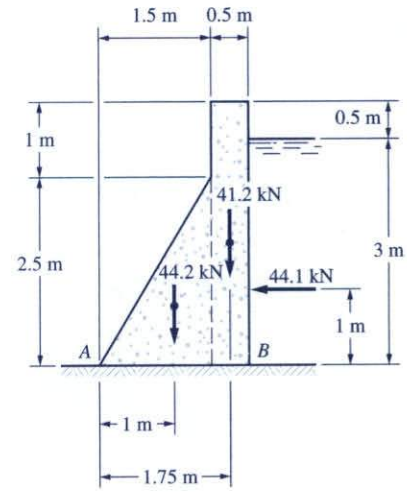


FIGURE P2–74

Section 2–11 Resultant of Distributed Line Loads

2–75 The loading on the bookshelf can be considered as three uniform loads, as shown in Fig. P2–75. Find the equivalent resultant force and specify its location along the shelf from point *A*.

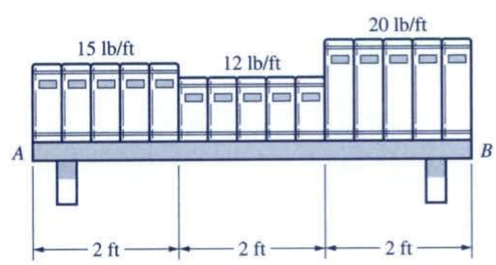


FIGURE P2–75

2–76 to **2–81** Replace the loading on the beams shown in Figs. P2–76 to Fig. P2–81 with an equivalent resultant force and specify their location along each beam measured from the left-hand end *A*.

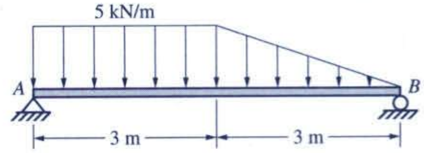


FIGURE P2–76

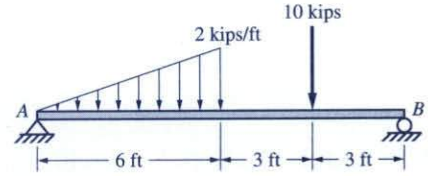


FIGURE P2–77

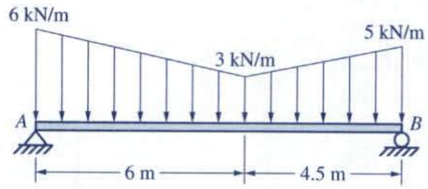


FIGURE P2–78

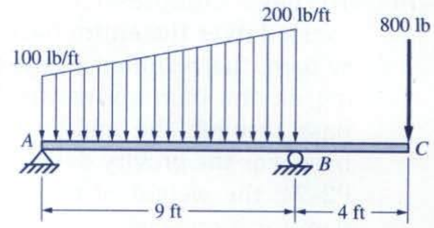


FIGURE P2–79

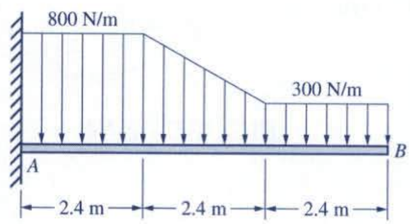


FIGURE P2–80

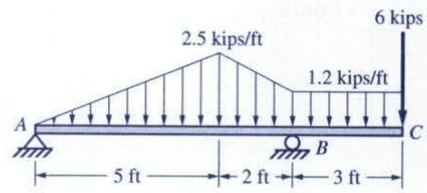


FIGURE P2–81

2–82 Replace the loading on the vertical post in Fig. P2–82 with an equivalent resultant force and specify its location along the post from the fixed end *A*.

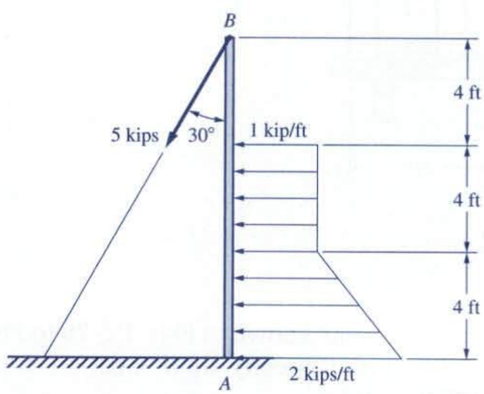


FIGURE P2–82

2–83 and **2–84** Replace the loading on the brackets in Fig. P2–83 and Fig. P2–84 with an equivalent resultant force and specify its location along *AB* measured from a convenient point.

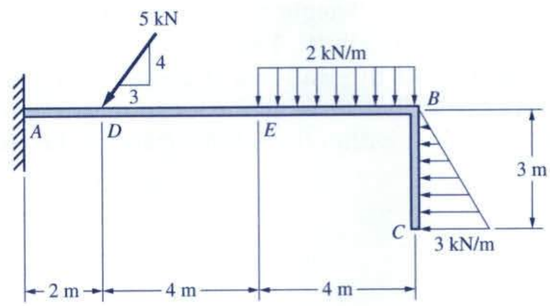


FIGURE 2–83

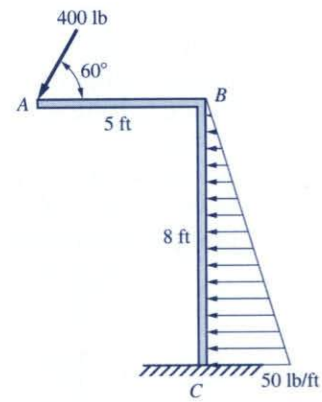


FIGURE P2–84

2–85 The beam in Fig. P2–85 is subjected to the distributed loading shown. Determine the distances *a* and *b* of the uniform load such that the resultant force and the resultant couple moment of the loading are zero.

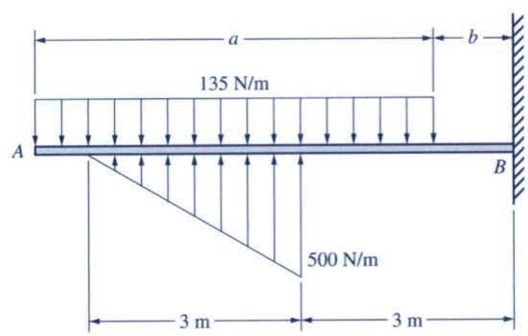


FIGURE P2–85

2–86 Determine the distances *a* and *b* of the triangular load in Fig. P2–86 so that the resultant force of the loading is a 200-lb force acting downward at the midpoint of the beam.

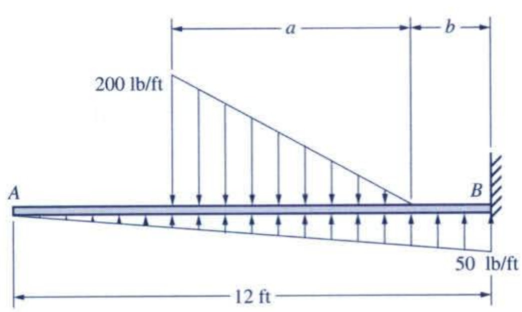


FIGURE P2–86

2–87 To have compressive soil pressure exerted over the entire base of a retaining wall, the resultant of the forces acting on the wall above the base must pass through the middle third of the base. The retaining wall in Fig. P2–87 is of concrete with a specific weight (weight per unit volume) of 150 lb/ft^3. The specific weight of the earth is 100 lb/ft^3. The lateral pressure exerted by the earth on a one-foot section of the wall is a triangular load as shown. Using a one-foot section of the wall, determine the location where the resultant passes through the base. Is it within the middle third of the base?

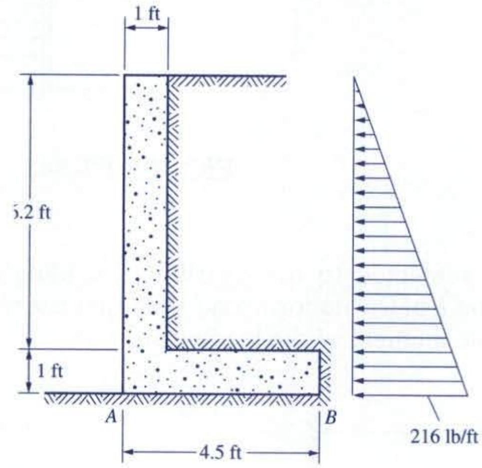


FIGURE P2–87

2–88 To have compressive soil pressure exerted over the entire base of a gravity dam, the resultant of the forces acting on the dam above the base must pass through the middle third of the base. The gravity dam in Fig. P2–88 is of masonry with a specific weight (weight per unit volume) of 23.6 kN/m^3. The lateral pressure of water on a one-meter section of the dam is a triangular load as shown. Using a one-meter section of the dam, determine the location where the resultant passes through the base. Is it within the middle third of the base?

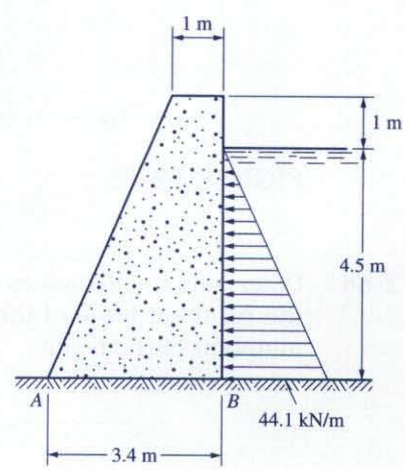
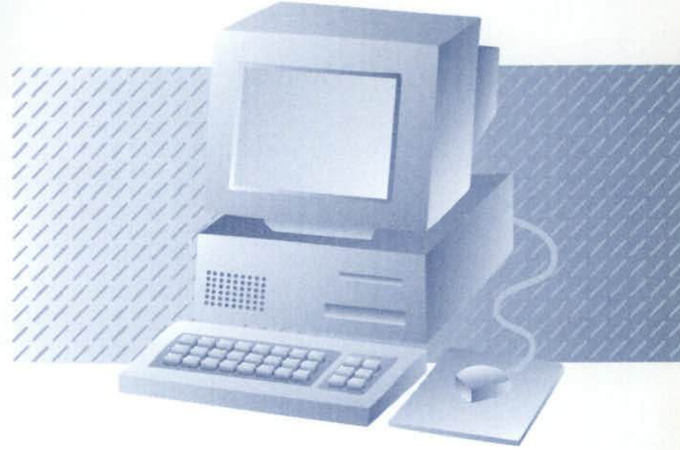


FIGURE P2–88

Computer Program Assignments



For each of the following problems, write a computer program using an appropriate programming language with which you are most familiar. Make the program user friendly by incorporating plenty of comments and input prompts so that the user will understand the input data to be entered and the limitations of their values. The output should include the data entered and the computed results, and they must be well labeled to identify each quantity. If a tabulated format is used, a proper heading must be included at the top of the table. Do not limit the program to any specific unit system. Indicate the consistent U.S. customary or SI units that can be used.

C2–1 Write a computer program that can be used to determine the resultant of a concurrent coplanar force system as shown in Fig. C2–1. The user input should include (1) the number of forces *n*, and (2) the magnitude and the direction angle in the standard position of each force F_i and θ_i. The output results must include the *x* and *y* components, and the magnitude and direction angle (in the standard position) of the resultant. Use this program to solve (*a*) Example 2–9, (*b*) Problem 2–25, and (*c*) Problem 2–26.

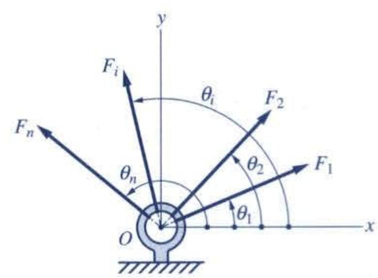


FIGURE C2–1

C2–2 Write a computer program that can be used to determine the resultant of a parallel coplanar force system as shown in Fig. C2–2. The user input should include (1) the number of forces *n*, and (2) the magnitude and location of each force F_i and d_i. Treat the downward force as positive and the upward force as negative. The output results must include the magnitude and direction of the resultant and its location along *AB*. Use this program to solve (*a*) Problem 2–60 and (*b*) Problem 2–61.

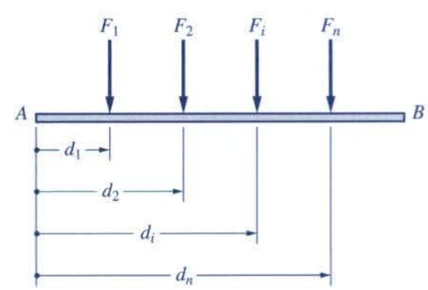


FIGURE C2–2